Guidance on using two-phase models for prediction the performance of a nanofluid minichannel heat sink

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ABSTRACT

High-performance heat sinks using nanofluids require Computational Fluid Dynamic (CFD) models that reliably calculate the cooling enhancement effect of the nanoparticles suspended in the base fluid. Whereas two-phase models can account for solid particles-fluid interaction, their use shows an unwanted variability in results, which often overestimate the heat transfer rate compared to single-phase models. This paper exposes the root cause of this variability and provides clear new guidance for the effective two-phase model use in minichannels. The guidance is grounded on an abstraction of the two-phase model physics that is physically consistent and coherent; this representation is a new statement in the nanofluid minichannel literature. In this representation, water serves as the first phase, while Newtonian nanofluid bubbles with equivalent thermal and physical properties ($\Phi < 100\%$) are used as the secondary phase as an alternative to the use of water as the first phase and solid nanoparticles as the secondary phase ($\Phi = 100\%$). This paper applies the Eulerian, mixture, and the Volume of Fluid (VoF) models to a rectangular minichannel over the Reynolds number range 100 to 2000, with 1% by volume Al_2O_3 nanoparticles in water. At the second-phase nanoparticle volume fraction concentration of $\Phi = 100\%$, all three models fail to provide accurate predictions, due to the models being used inappropriately, outside their physical application bounds. Improved results from all three models are obtained at second phase nanoparticle volume fractions lower than $\Phi = 10\%$ with good data collapse among the three models in the Nusselt number and friction factor. Mixture inhomogeneity in all models is limited to a thin film over the walls and this high localization is probably responsible for the insensitivity of the predictions on the two-phase model choice. This paper has therefore resolved the previous conflictual evidence on the use of steady two-phase models for minichannel conjugate heat transfer problems by highlighting a model implementation procedure by which consistent and coherent results can be obtained, which can take advantage of the mixture inhomogeneity capturing ability of these models.

Keywords: Minichannel heat sink, Nanofluid, Eulerian model, mixture model, Volume of fluid model, Conjugate heat transfer model.

1 INTRODUCTION

Modern electronic goods use ever smaller electronic units with higher heat flux. Unit thermal management turns out to be the key design challenge in many applications. Many studies have developed cooling methods to meet the cooling requirements in electronic units (Li *et al.*, 2019). Liquid cooled mini-micro channel heat sinks have been shown as one of the most useful cooling concepts for electronic heat sinks. Many studies have shown the excellent thermohydraulic efficiency of using liquids as working fluid in heat sinks instead of air in cooling high heat flux electronics. Moreover, the use of nanofluids as the working fluid in heat sinks has been studied to enhance heat transfer. Nanofluids can remove more heat per unit volume flow rate compared to ordinary liquids (Ali *et al.*, 2021b, Ali *et al.*, 2021a).

Research on nanofluid applications to thermal management is very active. Nanofluids can be modelled by a single-phase approach or with two-phase models. The single-phase approach assumes that small particles evenly disperse in the base fluid and that they have the same temperature as that of the base

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fluid. This method uses one artificial single fluid with thermal properties equivalent to those of the mixture. The thermal properties of the nanofluid mixture can be determined by correlations or from experiments (Sidik *et al.*, 2016). This single-phase fluid is simpler to model, requires less computational resources, and matches well against experimental results compared to a multi-phase mixture model (Santra *et al.*, 2009, Bhattacharya *et al.*, 2009). These advantages motivated many studies to use the single-phase model to simulate the nanofluid in mini/micro channel heat sinks. For instance, a study by Yue *et al.* (2015) explored the Al_2O_3 nanofluid performance in a microchannel heat sink. Their results show that using a higher volume fraction of nanoparticles leads to better cooling. Similarly. Ali *et al.* (2021a) used the single-phase approach to study Al_2O_3 nanofluid in a microchannel heat sink. They reported that Al_2O_3 nanofluids can enhance the Nusselt number and decrease the temperature of the heat sink base in comparison to pure water. These enhancements in the heat transfer were obtained at the expense of some increase in friction factor.

Two-phase models increase the fidelity of the simulation by dividing the nanoparticles and the base fluid into separate phases. Two-phase models are typically classed either as Eulerian-Lagrangian or Eulerian-Eulerian, based on how they handle the fluid and nanoparticle phases. The Eulerian-Eulerian two-phase models assume the working fluid and nanoparticles form two continuous fluid phases. These phases interact and the effect of this interaction on the state of each phase is computed using models such as the Volume Of Fluid (VOF), the mixture, and the Eulerian models. All three models are defined in the Eulerian-Eulerian framework in ANSYS (2017). Each model has its own way of accounting for mass, momentum, and energy conservation in each phase. The VOF model solves the momentum equations of phases that are not miscible. The VOF model solves the continuity equation within the secondary phases in order to track the volume fraction for the whole domain (Moraveji and Ardehali, 2013). The mixture model uses instead a single momentum equation and the energy equation to compute the overall mixture state. This model determines the volume fraction and the slip velocities for each phase (Ambreen *et al.*, 2021). The Eulerian model uses individual continuity, momentum, and energy equations for each phase. The volume fraction for every phase is solved as an additional scalar transport equation under the constraint that the sum of volume fractions adds up to one.

In the numerical investigation of nanofluids, the effectiveness of single phase and two-phase models have been studied by several researchers. Choosing which one is the most accurate model has been a prominent issue of debate. Many studies provided evidence for the accuracy of the single-phase model, such as studies by Akbari *et al.* (2012), Albojamal and Vafai (2017) and Keshavarz Moraveji and Esmaeili (2012). These studies reported that the thermal prediction of single-phase computations is in good agreement with experimental results. Other researchers reported the importance of the additional physical fidelity provided by the two-phase models, arguing that the two-phase models show their ability to offer more accurate predictions by considering the interaction between the base fluid and solid nanoparticles. The effectiveness of using the two-phase models has been reported by numerous researchers, for example, Moraveji and Ardehali (2013) used VOF, the mixture and the Eulerian models in a minichannel heat sink. Kalteh *et al.* (2012) used the Eulerian model in a microchannel heat sink and Ghale *et al.* (2015) used VOF and mixture models in a microchannel heat sink.

Despite the numerous numerical studies that investigated the nanofluid using single phase and two-phase models, there is still work that can be done to understand the behaviour of two-phase models when simulating nanofluids. For example, several studies have reported that two-phase models often overestimate the heat transfer rate in comparison to the single-phase model. This two-phase model overestimation in heat transport is stated, for instance, in Akbari *et al.* (2012), Mojarrad *et al.* (2013), Hejazian and Moraveji (2013), and in Kalteh *et al.* (2012).

Ali *et al.* (2022a) show how the two-phase mixture model can be used to obtain thermal predictions consistent with the ones from the experiment and from a single-phase model in a minichannel heat sink over a large range of Reynolds numbers. This was achieved by implementing what they refer to as a physically consistent protocol for the definition of the fluid phases in the simulation. They used water

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for the first phase and assumed the second phase was made of Newtonian nanofluid bubbles that were dispersed in the first phase. These bubbles had a second-phase volume fraction concentration of nanoparticles Φ in them. The volume fraction concentration of the nanofluid bubbles in the mixture was α_{nf}. This gave a volume fraction concentration of the nanoparticles in the mixture of $\phi = \alpha_{nf} \times \Phi$. This representation enabled Ali *et al.* (2022a) to state the volume fraction concentration of the nanofluid bubble α_{nf} and the second-phase volume fraction concentration of the nanoparticles within them Φ as two separate variables, which were multiplied together to yield the nanoparticle volume fraction ϕ in the nanofluid mixture. This research explored various (α_{nf},Φ) combinations and determined how predictions of heat transfer rate and of friction factor compare to the ones from a single-phase simulation, in which a homogeneous fluid of equivalent thermophysical properties was used. This comparison identified bounds for α_{nf} and Φ beyond which significant heat transfer overestimates with the two-phase mixture model occurred. Using this insight, Ali *et al.* (2022a) produced mixture model thermal predictions that consistently followed the ones from the single-phase model and experimental measurements.

To date, no study has been undertaken on how to prevent the heat transfer rate overestimation in nanofluid simulations that use either the Volume of Fluid model or the Eulerian model. This paper addresses this specific knowledge gap by extending the work of Ali *et al.* (2022a) to the Volume of Fluid model and to the Eulerian model for two-phase flows. This paper makes the following contributions to the state of the art:

- It determines whether it is appropriate to represent a two-phase heat sink nanoflow as bubbles of nanofluid dispersed in water in the Volume of Fluid and in the Eulerian models, as Ali *et al.* (2022a) did with the two-phase mixture model.
- It exposes the consequence of adopting the bubbles of nanofluid representation in Ali *et al.* (2022a) in the Volume of Fluid, mixture, and Eulerian models on the thermal and hydraulic performance predictions.
- It compares the thermal and hydraulic predictions obtained by the nanofluid representation from Ali *et al.* (2022a) to the ones obtained by the simpler representation of a second phase made of nanoparticles that has often been used for two-phase models and to predictions using a single-phase model. By this, it shows that the upper bound limitation to the nanoparticle volume fraction in the mixture model second phase reported by Ali *et al.* (2022a) likewise applies to the Volume of Fluid (VOF) and to the Eulerian models.

Through an in-depth exploration of these models, this paper augments the thermodynamic community's understanding of how to use two-phase models in nanofluid simulations and fortifies the overall predictive capabilities of the two-phase models. The context of this work is a minichannel heat sink is subjected to a heat flux of 16.67 W/cm² with Al_2O_3 nanofluid cooling at 303.15 K, and the Reynolds numbers range from 100 to 2000.

2 METHODOLOGY

2.1 Mathematical formulation

In this investigation, a numerical analysis is conducted to explore the phenomenon of Al_2O_3 nanofluid cooling in a minichannel heat sink under laminar flow conditions. The heat sink and computational domain are outlined in Figure 1. The heat sink base receives a constant heat flux, which serves as the boundary condition. By solving the Laplace equation, the steady heat conduction through the solid walls of the minichannel is determined.

$$\nabla^2 T_s = 0 \tag{1}$$

Equation (1) is employed to model the temperature distribution T_s within the solid surfaces, while the time-averaged Navier-Stokes equations are utilized to characterize the behavior of the coolant, as established by previous works (Ali *et al.*, 2021b). The fluid flow is defined as a Newtonian fluid and its thermal properties are determined either by the single-phase model or by a two-phase model.

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2.2 Single-phase model

The nanoparticles and the fluid are simulated as a single uniform Newtonian fluid by the single-phase model. The numerical solution involves solving the continuity, momentum, and energy equations based on the following assumptions:

1. The fluid is characterized as three-dimensional, laminar, Newtonian and incompressible.
2. The flow is steady, with temperature dependence, and a uniform flow velocity at the inlet.
3. Radiative heat transfer and gravity effects are neglected.

The governing equations for the continuity, momentum, and energy equations are given in (Ali *et al.*, 2021b) following (Ambreen and Kim, 2017). Within, ρ_{nf} is the density, μ_{nf} is the molecular viscosity, k_{nf} is the thermal conductivity, $\vec{u}_{nf}$ is the velocity vector, p_{nf} is the absolute pressure, and T_f is the absolute temperature of the nanofluid.

2.3 Two-phase models

Two-phase models treat the flow as two continuous phases, using phase volume fractions to define their presence. This work uses the Volume Of Fluid (VOF), the mixture, and the Eulerian two-phase models. Following Ali *et al.* (2022a), for all three mixture models:

1. Phase one is water and phase two is a uniform Newtonian fluid (suspension of nanoparticles in water).
2. The flow is steady, laminar, incompressible, and three-dimensional.
3. The thermal properties of the working fluid vary with temperature.
4. The gravitational effect is considered, while radiative heat transfer is neglected.
5. The volume fraction (Φ) of nanoparticles in the second phase is so low that collisions and heat conduction between nanoparticles are negligible.

For the mixture and Eulerian models:

1. The second phase consists of uniformly sized spherical bubbles.
2. The volume fraction (α_{nf}) of bubbles is low enough to prevent a direct momentum exchange between them.
3. The nanoparticle diameter is smaller than the bubble diameter, ensuring the value of Φ stays smooth continuous inside the bubble, and bubble sphericity remains unaffected by the presence of the nanoparticles.
4. The bubble diameter is smaller than the minichannel hydraulic diameter, ensuring the smooth and continuous nature of α_{nf}.

2.3.1 Volume of fluid model

The two-phase VOF model is best suited to flows made up of two immiscible fluids separated by a trackable interface. Here are the key assumptions of this model (Habeeb *et al.*, 2022):

1. The model tracks the volume fractions for both phases by solving the continuity equation for the base fluid (water) and using the constraint that the sum of volume fractions for all phases equals unity.
2. A single momentum equation is used to find the velocity field that is shared by all phases.
3. Physical properties are computed by considering a weighted mean of various phases, with weights found by their respective volume fractions within each control volume.

2.3.2 Mixture model

The two-phase mixture model treats the phases as interpenetrating continua. A single momentum equation is used to find the velocity of the mixture and the relative velocities of each phase is determined analytically. This versatile model finds application in various scenarios, bubbly flows, sedimentation processes, and cyclone separators (ANSYS, 2017). The nanofluid is represented as a two-phase mixture under the following assumptions:

1. Both phases within the system experience identical pressure conditions.
2. The two phases are in thermal equilibrium with one another and a single mixture temperature transport equation is solved.

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3. Each phase has its distinct velocity field and a specific proportion of each phase is present in a defined control volume.
4. The secondary dispersed phase is made of spherical bubbles of uniform size.
5. The motion of the secondary phase relative to the base fluid (water) is governed by the drag law by Naumann and Schiller (1935) and phase slip determines the phase concentration.

The continuity equation, mixture momentum equation, drift velocities, drag equation, and the scalar transport equation for the mixture temperature are given in Ali *et al.* (2022a).

2.3.3 Eulerian model

The Eulerian multiphase model is the most advanced model among the three two-phase models because it resolves both momentum and heat transfer phase interactions. The flow is made of two-phases with the following features (Habeeb *et al.*, 2022):

1. Both phases within the system experience identical pressure conditions.
2. Phase one (water) and phase two (nanofluid bubbles) are both treated as continuous media.
3. Mass, momentum, and energy conservation are separately applied to each phase.
4. The total volume fractions add up to one.
5. Temperature and velocity differences between the phases are resolved.

Source terms $F_{D,bf}$ and $F_{D,nf}$ in the momentum conservation of each phase couple the motion through a coefficient of momentum exchange I based on the second phase relaxation time T_{nf}, which is determined by the nanofluid bubble drag coefficient f_D estimated according to Naumann and Schiller (1935). The heat transfer phase interaction is determined by the volumetric interphase heat transfer coefficient (h_v), obtained from the second phase Nusselt number Nu_{nf} from Ranz (1952).

2.4 Homogeneous nanofluid equations

To simulate the Al_2O_3 nanoparticles in the working fluid with volume fraction ϕ=1%, two distinct methods are employed. Table 1 provides the thermal properties of the working fluid, nanoparticles, and the solid domain. The first method is the single-phase approach, which models the Al_2O_3 nanoparticles and working fluid as one homogeneous flow with equivalent thermal and physical properties. The second method is the two-phase approach, which involves using water as phase one and Newtonian nanofluid bubbles as phase two. Different combinations of the phase two volume fraction α_{nf} and of the volume concentration Φ of Al_2O_3 nanoparticles in the second phase are tested, as outlined in Table 2. Each combination results in the same overall mixture with a 1% volume fraction of Al_2O_3 nanoparticles.

Table 1**:** The thermal properties of water, nanoparticles, and Aluminium.

	ρ (kg/m^3)	C_P (J/kg · K)	k (W/m · K)	μ (N · s/m^2)	Diameter of nanoparticles (nm)
Water	998.2	4182	0.613	0.000855	-
Al_2O_3	3880	733	36	-	50
Aluminium	2719	871	202.4	-	-

The nanofluid density ρ_{nf}, constant pressure heat capacity Cp_{nf}, and thermal conductivity k_{nf}, are obtained from the second phase Al_2O_3 nanoparticle volume fraction ϕ weighted average (Xuan and Roetzel, 2000) and the dynamic viscosity $\mu_{nf} = \mu_{bf}(123\phi^2 + 7.3\phi + 1)$ (Ali *et al.*, 2021b).

Table 2**:** Combinations of α_{nf} and Φ that yield the same ϕ value of 1% for the Al_2O_3 nanofluid mixture.

Second phase volume fraction α_{nf}	0.5	0.3	0.2	0.1	0.08	0.06	0.04	0.02
Second phase Al_2O_3 Nanoparticle volume fraction Φ	0.02	0.0333	0.05	0.1	0.125	0.1667	0.25	0.5

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2.5 Numerical solver

The minichannel geometry shown in Figure 1 is modelled by ANSYS FLUENT 19.5 commercial software. At the minichannel inlet, a uniform and normal velocity (u_{in}) is specified along with a uniform temperature of 303.15 K. The inlet volume fraction of the second phase is determined based on Table 2. The minichannel outlet has a time-invariant uniform outlet gauge pressure of zero. The base wall is subjected to a constant heat flux of 16.67 W/cm^2 while all other exterior walls have zero heat flux. In the single-phase model, constant values for density (ρ_{nf}), specific heat capacity (Cp_{nf}), dynamic viscosity (μ_{nf}), and thermal conductivity (μ_{nf}) are utilized. These values are determined at the beginning of the calculation by ϕ weighed averaging (c.f. §2.4) with $\phi = 1\%$. For the two-phase models, constant values of density (ρ_{bf}), specific heat capacity (Cp_{bf}), dynamic viscosity (μ_{bf}), and thermal conductivity (k_{nf}) are employed for the first phase, while the second phase adopts the same constant properties definition as the single-phase model. These properties are determined by ϕ weighed averaging (c.f. §2.4) using the corresponding Φ values from Table 2. It is noteworthy that these constant properties are consistent between single phase and two-phase simulations in that they all model the same mixture overall phase fraction $\phi = 1\%$. The ANSYS FLUENT 19.5 software uses a double-precision steady pressure-based Coupled flow solver. The pressure calculations employ PRESTO! scheme, while the momentum and energy equations use the second-order upwind method. The volume fraction equation is solved using the QUICK scheme. The results are deemed as converged once the residuals for governing equations fall under 1×10^{-6}. Each simulation takes approximately 15-minutes to compute on a 16-core shared memory computer cluster with 3.2 GHz cores.

2.6 Geometry

Figure 1 shows the isometric view of the minichannel heat sink, which has nine parallel rectangular-shaped ducts, each characterized by specific dimensions. Each duct measures 30 mm in length, 4 mm in height, 1 mm in channel width, 1 mm in fin width, and 3 mm in channel height. To simplify the computational process, only one duct is considered in this investigation. Symmetry is assumed to exist in the temperature field between adjacent ducts, as depicted by the dashed lines in Figure 1. Table 3 provides the thermo-physical properties of aluminum, which is used to make the heat sink. The Reynolds number (Re) serves as a reference and ranges from 100 to 2000. It is calculated using the following equation (Ali, 2023):

$$Re = \frac{\rho_{nf} u_{in} D_h}{\mu_{nf}} \tag{2}$$

The hydraulic diameter (D_h) is defined by the equation (3):

$$D_h = \frac{4A}{P} = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b} \tag{3}$$

in this equation, P, a, and b refer to the perimeter, width, and thickness of the duct.

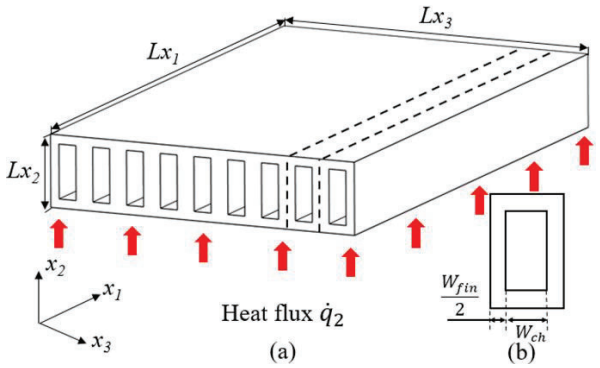

Figure 1: (a) The minichannel heat sink and (b) one duct dimensions.

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2.7 Data analyses

The Nusselt number (Nu) is calculated according to equation (4):

$$Nu = \frac{hD_h}{A_b(T_s - T_{mean})} \tag{4}$$

here h is the average heat transfer coefficient, A_b is the base area of the heat sink, T_s is average base wall temperature, and T_{mean} is the mean fluid temperature (Ali *et al.*, 2022a).

The friction factor (f) is calculated according to equation (5) (Ali *et al.*, 2022b).

$$f = \frac{2D_h\Delta p}{\rho_{nf}u_{in}^2 Lx_1} \tag{5}$$

2.8 Numerical mesh

The rectangular geometry of the microchannel enables the use of a simple hexahedral computational mesh. To assess the impact of spatial mesh refinement on the predictions, a comparative analysis was conducted using six distinct meshes, between 0.275 to 2.2 million elements. The computational accuracy investigation focused on evaluating two key parameters: the average temperature of the bottom wall and the pressure drop. These evaluations were carried out at an inlet temperature of 303.15 K, $\phi = 1\%$ of Al_2O_3 nanoparticles, at the Reynolds number of 274. Both the single-phase model and the two-phase mixture model were tested. The two-phase model was run with ($\alpha_{nf} = 10\%, \Phi = 10\%$) and ($\alpha_{nf} = 1\%, \Phi = 100\%$). The predictions achieved a significant level of independence from the spatial mesh size beyond 1.1 million elements. Therefore, the computational mesh consisting of 2.2 million cells was selected for the next stages of the investigation. Figure 2 provides a visual representation of the mesh of this chosen mesh.

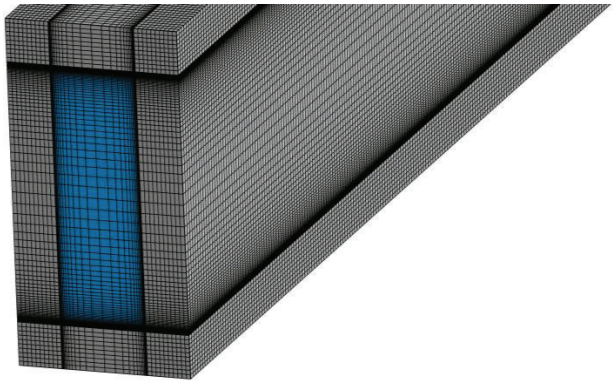

Figure 2: The mesh structure of the heat sink.

2.9 Validation

This study performed a validation against experimental data acquired from Kumar and Sarkar (2019). Figure 3 (a) presents a comparative assessment of the average heat transfer coefficient, employing the single-phase model with water. The predictions show a fair level of agreement against the experimental results, with a maximum deviation of 16.7% observed at a Reynolds number of 92. Figure 3 (b) illustrates the friction factor by the single-phase results of water flow in the minichannel and the correlation of friction factor for laminar flow through the square duct, ($f = 62.2\ Re^{-1}$) (Cengel, 2010). This correlation pertains to a fully developed region in laminar flow. The value of f is compared by considering the pressure drop from the end of the hydrodynamic entrance region and the outlet of the microchannel. The length of the hydrodynamic entrance region (L_h) was approximated using $L_h \cong 0.05ReD_h$ (Cengel, 2010). The single-phase prediction is in good agreement with the friction factor correlation in terms of trend and values. For the two-phase mixture model with water employed in both phases, the average heat transfer coefficient and friction factor predictions closely match those obtained from the single-phase model across the entire Reynolds number range, spanning from 92 to 554. These results provide evidence of the appropriateness of the models for predicting the thermal and hydraulic characteristics of the heat sink.

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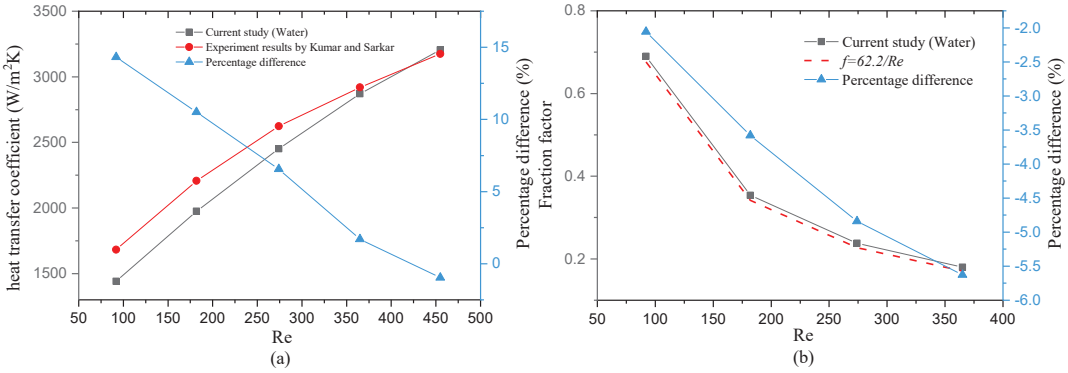

Figure 3**:** (a) the average heat transfer coefficient and (b) friction factor.

3 RESULTS AND DISCSSION

3.1 Influence of the volume fraction of the second phase α_{nf} on the thermal characteristics

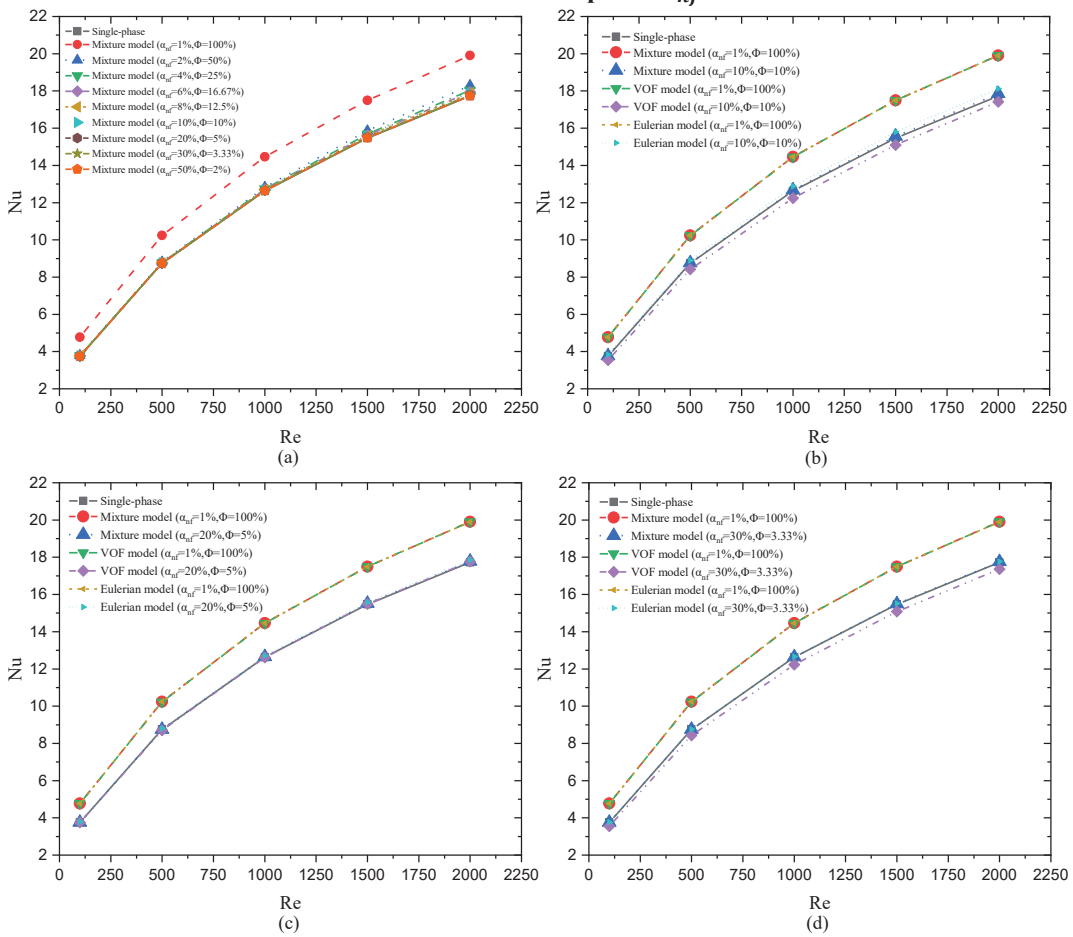

Figure 4**:** Nusselt number of the heat sink at $100 \leq Re \leq 2000$ using the single-phase model approach compared to (a) two-phase mixture model $1\% \leq \alpha_{nf} \leq 50\%$, (b) two-phase mixture, VOF and Eulerian models at $\alpha_{nf} = 1\%$ and $\alpha_{nf} = 10\%$, (c) two-phase mixture, VOF and Eulerian models at $\alpha_{nf} = 1\%$ and $\alpha_{nf} = 20\%$ and (d) two-phase mixture, VOF and Eulerian models at $\alpha_{nf} = 1\%$ and $\alpha_{nf} = 30\%$.

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Figure 4 (a) shows the Nusselt number predicted by the single-phase model and by the mixture model over the range of Reynolds numbers between 100 to 2000. This extends similar results by Ali *et al.* (2022a) that covered the Reynolds number from 92 to 455. The mixture model with (α_{nf} = 1%, Φ = 100%) overestimates the Nusselt number, owing to the incorrect representation of the interaction physics between nanoparticles and the base fluid when a phase is exclusively made of nanoparticles, consistent with the findings reported by Akbari *et al.* (2011), Moraveji and Ardehali (2013), Mojarrad *et al.* (2013), and Ali *et al.* (2022a). As the second-phase nanoparticle volume fraction is reduced below Φ = 50%, the mixture model Nu predictions progressively align with the single-phase model ones, with an appreciable overlap as the second phase volume fraction α_{nf} raises above 10%. The nanoparticles are designed to achieve good dispersion in the base fluid and having a second phase volume fraction higher than 10% enables the model to better represent the mixing out of the second phase throughout the domain. This finding is consistent with the advice in (ANSYS, 2017) on using second-phase volume fraction > 10% in mixture model simulations.

Figure 4 (b-d) compares the Nusselt number predictions among the single-phase and the VOF, mixture and Eulerian two-phase models at different α_{nf}. All two-phase models at ($\alpha_{nf} = 1\%, \Phi = 100\%$) predict Nusselt number that is clearly above that of the single-phase prediction, in accordance with the findings reported by Akbari *et al.* (2011), Moraveji and Ardehali (2013). Reducing Φ below 10% (and increasing α_{nf} above 10%) aligns the Nusselt number predictions from the Euler and mixture models with the ones of the single-phase, with a notably good monotonic collapse among the predictions as α_{nf} is increased from 10% to 30%. This alignment in heat transport predictions between the single-phase model and two-phase models stands as a significant result of this study, demonstrating the potential to address concerns regarding the application of the two-phase models to minichannel heat transport problems through thoughtful model implementation.

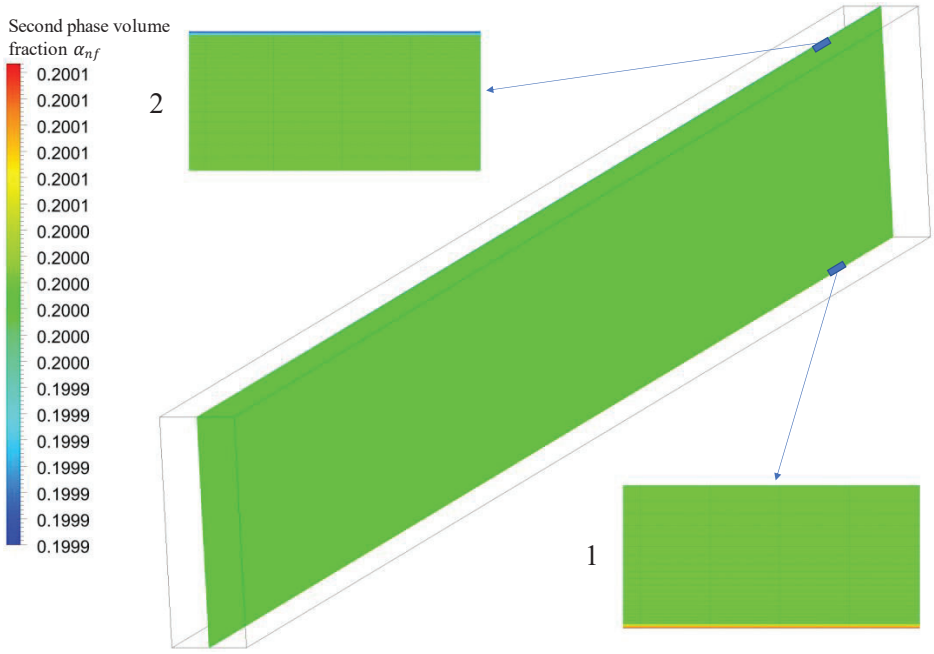

Figure 5: The second volume fraction in n the cross-sectional in the x_3 plane usimg Mixture model at ($\alpha_{nf} = 20\%, \Phi = 5\%$) under Re=100.

The VOF predictions do not exhibit the same asymptotic behaviour with α_{nf} increasing from 10% to 30% as the Euler and mixture models, as shown by the Nusselt number at $\alpha_{nf} = 30\%$ becoming underpredicted compared to all other models. Figure 5 provides a possible explanation for this VOF

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model behaviour. The second phase volume fraction across the channel mid-span, predicted by the mixture model, shows that the second phase is almost uniformly distributed across the computational domain. The insets 1 and 2 in Figure 5 show minimal phase stratification at the top and bottom walls, confined to the first interior cell. Whereas the mixture model and the Eulerian model are designed for modelling interpenetrating fluids, the VOF is designed for modelling immiscible fluids in which a traceable interface forms. The small streamwise length of the minichannel does not allow any substantial phase separation to take place and no interface is formed between the two phases, causing the VOF model to be used outside its intended operating scope.

3.2 Influence of the volume fraction of the second phase α_{nf} on the hydraulic characteristics

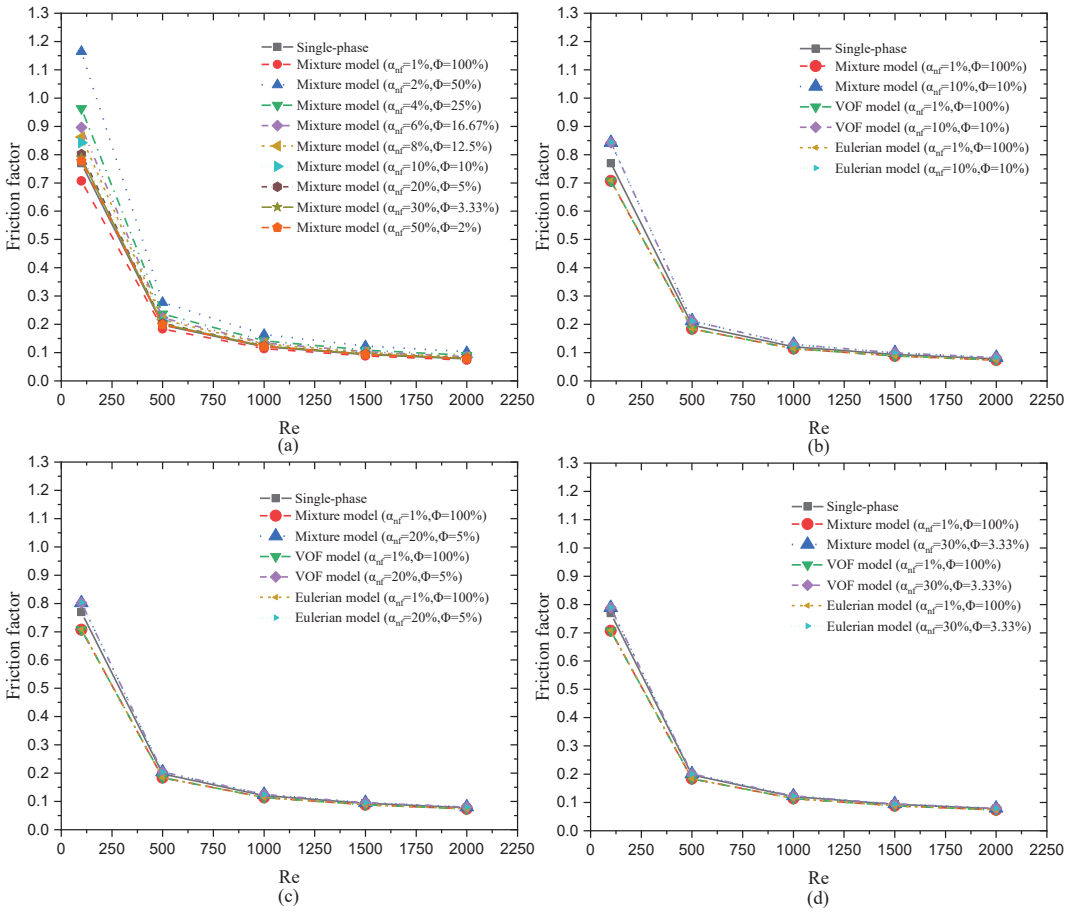

Figure 6: Friction factor at $100 \leq Re \leq 2000$ using the single-phase model approach compared to (a) two-phase mixture model $1\% \leq \alpha_{nf} \leq 50\%$, (b) two-phase mixture, VOF and Eulerian models at $\alpha_{nf} = 1\%$ and $\alpha_{nf} = 10\%$, (c) two-phase mixture, VOF and Eulerian models at $\alpha_{nf} = 1\%$ and $\alpha_{nf} = 20\%$ and (d) two-phase mixture, VOF and Eulerian models at $\alpha_{nf} = 1\%$ and $\alpha_{nf} = 30\%$.

Figure 6 characterises the minichannel hydraulic performance in terms of the friction factor predicted by the same combination of models at Figure 4. In all two-phase models where $\Phi = 100\%$, the second phase is made up entirely of nanoparticles, while where $\Phi < 100\%$, the second phase is made up of Newtonian nanofluid bubbles. The latter is the more correct model, as two-phase models use Newtonian viscosity in their formulation. In Figure 6 (a-c) an underestimate of the friction factor for all three two-phase models in comparison to the single-phase model is observed when using $(\alpha_{nf} = 1\%, \Phi = 100\%)$. Using $\Phi = 100\%$ gives an asymptotic convergence in the two-phase models' predictions towards the single-phase model one, as shown in Figure 6 (a). For all three two-phase models, this

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collapse in α_{nf} increases from 10% to 30%. Rα_{nf} and leads to a steep increase in friction factor. These findings highlight the challenges associated with modelling nanofluids.

4 CONCLUSIONS

The debate of whether single-phase or two-phase models of minichannel and microchannel nanofluid heat sinks provide better predictions has long affected the confidence of the numerical modelling community. This study places a firm answer by identifying model usage pitfalls and showing how the judicious definition of nanofluid minichannel test cases provides consistent predictions.

Tangible progress in flow modelling practice is provided by defining guidance on the use of two-phase models within appropriate applicability bounds, for the Euler, mixture, and Volume of Fluid models.

In a two-phase model approach involving water as the first phase and solid nanoparticles within the second phase (fluid-solid), all three two-phase models overestimate the Nusselt number when compared to the single-phase model. This paper shows that the overestimate is systematically corrected by avoiding solid particles as the second phase, which is re-defined as made of Newtonian nanofluid bubbles.

With the second phase so redefined, consistent Nusselt number predictions are obtained at second phase volume fractions $\alpha_{nf} > 10\%$ and an appreciable data collapse among the Eulerian, mixture, and single-phase models is achieved with a second-phase volume fraction $\alpha_{nf} \leq 10$. This, in principles, provides researchers with the flexibility of using whatever two-phase model out of these two they can access for modelling minichannel mass and heat transport.

An important observation is that the Volume of Fluid two-phase model, while still retaining good predictive ability, does not deliver the same prediction overlap as the other two models. The design feature of nanoparticles as highly dispersing prevents phase stratification and the formation of a traceable phase interface within the short streamwise length of the microchannel, for the VOF to work. The use of a second phase volume fraction and of the second-phase nanoparticle volume fraction within the above stated bounds also provide the same desirable hydraulic performance prediction overlap among the models.

The guidance on two-phase model use provided in this paper, in the context of a nanofluid cooled minichannel, is in principles extensible to microchannel heat management systems for the microelectronic industry, potentially providing higher fidelity models with lower thermal performance error bounds. This underpins scientific research and development of high heat flux electronic devices that are key enablers for a sustainable power generation infrastructure.

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