

A MULTI-OBJECTIVE OPTIMIZATION STRATEGY FOR DISTRICT HEATING PRODUCTION PORTFOLIO PLANNING

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ABSTRACT

The imperative to decarbonize energy systems has intensified the need for efficient transformations within the heating sector, with a particular focus on district heating networks. This study addresses this challenge by proposing a comprehensive optimization approach evaluated on the district heating network of the Märkisches Viertel of Berlin. Our objective is to simultaneously optimize heat production with three targets: minimizing costs, minimizing CO₂-emissions, and maximizing heat generation from Combined Heat and Power (CHP) plants for enhanced efficiency.

To tackle this optimization problem, we employed a Mixed-Integer Linear Program (MILP) that encompasses the conversion of various fuels into heat and power, integration with relevant markets, and considerations for technical constraints on power plant operation. These constraints include start-up and minimum downtime, activation costs, and storage limits.

The ultimate goal is to delineate the Pareto front, representing the optimal trade-offs between the three targets. We evaluate variants of the ϵ -constraint algorithm for their effectiveness in coordinating these objectives, with a simultaneous focus on the quality of the estimated Pareto front and computational efficiency. One algorithm explores solutions on an evenly spaced grid in the objective space, while another dynamically adjusts the grid based on identified solutions. Initial findings highlight the strengths and limitations of each algorithm, providing guidance on algorithm selection depending on desired outcomes and computational constraints.

Our study emphasizes that the optimal choice of algorithm hinges on the density and distribution of solutions in the feasible space. Whether solutions are clustered or evenly distributed significantly influences algorithm performance. These insights contribute to a nuanced understanding of algorithm selection for multi-objective multi-energy system optimization, offering valuable guidance for future research and practical applications for planning sustainable district heating networks.

1 INTRODUCTION

District heating networks are anticipated to be instrumental in decarbonizing the heating sector, particularly in urban and suburban regions. The allocation of resources towards non-fossil fuel-based generation technologies in these networks enables the reduction of carbon emissions associated with heat production in numerous households. Concurrently, this transformation leads to a more dispersed distribution of heat generation, resulting in an increasingly intricate challenge in terms of operational optimization. Moreover, optimization efforts not only prioritize economic objectives but also emphasize

^a The preparation of this paper has been overshadowed by Jan-Patrick's death. We had intended to write jointly: most of the main ideas were worked out together, and we have done our best to complete them. In sorrow, we dedicate this work to his memory.

ecological objectives and the enhancement of sector coupling efficiency (Dorotić *et al.*, 2019; Falke *et al.*, 2016).

A good solution for a problem with conflicting objectives is one where you cannot improve on one of the objectives without worsening at least one of the others, i.e., Pareto optimal solutions. In *Pareto optimization*, the goal is to find such a set of solutions. Various scholarly works have focused on optimizing the design and operation of district heating networks, and more generally, distributed energy systems. The most commonly utilized approach involves the application of linear programming or mixed-integer linear programming (MILP) formulations (Fazlollahi *et al.*, 2012; Bracco *et al.*, 2013; Wu *et al.*, 2016; Shukla and Singh, 2016; Buoro *et al.*, 2013; Morvaj *et al.*, 2016; Weinand *et al.*, 2019; Chen *et al.*, 2023). A realistic model of urban district heating networks can be highly intricate due to the involvement of numerous resources, energy flows, and interconnections within the system, as well as a diverse set of constraints.

To achieve multi-objective optimization of such systems, researchers have employed a variety of optimization techniques, including genetic and evolutionary algorithms (Fazlollahi *et al.*, 2012), as well as linear programming with a weighted sum approach on the objective functions (Wu *et al.*, 2016; Shukla and Singh, 2016; Buoro *et al.*, 2013; Bracco *et al.*, 2013) and ϵ -constraint algorithms (Morvaj *et al.*, 2016; Fazlollahi *et al.*, 2012) or a combination of both (Dorotić *et al.*, 2019). However, the performance of these optimization algorithms often raises concerns. Evolutionary algorithms require more computational effort than ϵ -constraint algorithms or integer cut constraints (Fazlollahi *et al.*, 2012). Combined algorithms similarly involve a large number of calculations to obtain the Pareto front with an acceptable level of detail (Dorotić *et al.*, 2019). Note that suitable metrics to evaluate the quality of an estimation of the Pareto front is yet another research question (Datta and Figueira, 2012). Due to the increasing size of district heating networks and increasing complexity due to distributed energy/heat generation, we are interested in finding algorithms that provide the best trade-off between computational effort and the level of detail of the representation of the Pareto front. Finally, district heating operators rely on tools that explore solutions on the Pareto front that cover a range of flexibility options that are within a reasonable operational range and can be computed efficiently to support studies on several (investment) scenarios.

This paper contributes to the field of multi-objective production portfolio optimization in district heating systems, emphasizing two case studies, a medium-sized city and the full Berlin model, representing the most complex Western European district heating system. We evaluate the efficiency of algorithms searching static/dynamic grids in the solution space to generate meaningful solutions on the Pareto front and present the pros and cons of the different algorithmic strategies concerning multi-objective optimization of district heating network operations. Finally, we demonstrate how such analyses can support system operators in exploring the flexibility of operations with respect to three objectives. Thereby, we aim to enhance decision-making processes and support the development of optimal strategies for district heating systems. In the subsequent sections of this paper, we will delve into the mathematical model, introduce the coordination algorithms, present computational results from two case studies, and draw meaningful conclusions on the structure of these problems. Our ultimate goal is to improve the understanding of multi-objective optimization techniques and their application to real-world district heating systems.

2 MATHEMATICAL MODEL

The overall goal of production portfolio optimization for a district heating network is to fulfill the heat demand of its customers at minimum cost and CO₂-emissions while maximizing efficiency. Figure 1 illustrates the resource flows and conversion between them. Various types of generation technologies transform fuel into heat and power. The produced heat can be used to cover the demands of different districts or can be conserved in short-term or seasonal heat storage. Power can be utilized for heat circulation pumps or electric heat generators such as heat pumps or electric heaters. It can either be purchased or co-generated by CHP-plants. Co-generated excess power can be sold.

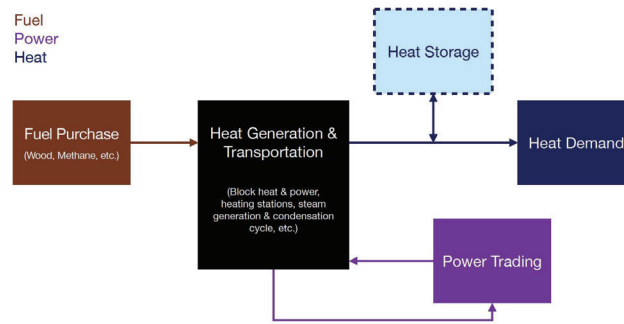


Figure 1: The multi-energy system comprises the dynamic flow of fuel (brown), power (purple), and heat (dark blue). Various technological blocks engage in conversion processes, while market blocks function as sources or sinks of resources.

We consider the energy production portfolio optimization problem as a mixed-integer linear program (MILP). The problem represents a *unit-commitment-problem* with respect to a multi-energy-system. Let I be the set of power plants in the district heating system using the set of resources R , and T be the time horizon for which unit commitment is analyzed.

Three *objectives* are optimized with decreasing priority: Economic costs are *minimized*:

$$\min_z \sum_{t \in T} \sum_{r \in R} C_t^r P_t^r + \sum_{i \in I} \sum_{t \in T} C_{i,t} z_{i,t} + \sum_{i \in I} \sum_{t \in T} \sum_{r \in R} \omega_{i,t}^r (Qin_{i,t}^r, z_{i,t}) - \sum_{t \in T} \sum_{r \in R} S_t^r E_t^r \quad (1)$$

where $z_{i,t} \in \{0,1\}$, being the operation variable of power plant i at time step t . For resource r , time step t , and power plant i , cost coefficients are given by deficit purchasing costs related to the price of purchasing C_t^r and the purchased amount P_t^r , operational costs comprising $C_{i,t}$, subtracted by the revenues collected from selling the excess amount of the produced resource E_t^r at the selling price S_t^r . Herein, variable costs $\omega_{i,t}^r$ are modeled as a piecewise linear function covering the maintenance, transportation, taxes, and running costs with respect to $Qin_{i,t}^r$, the total amount of resource r flowing into power plant i at time step t . CO₂-emissions are *minimized*:

$$\min_z \sum_{i \in I} \sum_{t \in T} Emissions_{i,t}, \text{ with } Emissions_{i,t} = f_i \delta_t \sum_{r \in R} Qin_{i,t}^r z_{i,t}, \quad (2)$$

where f_i is the CO₂-emissions factor at a power plant i and δ_t the length of time step t . As a measure of energy efficiency CHP output heat is *maximized*:

$$\max_z \sum_{i \in I'} \sum_{t \in T} Qout_{i,t}^{heat} z_{i,t}, \quad (3)$$

where $Qout_{i,t}^{heat}$ is the heat produced in CHP plants $I' \in I$.

Constraints arise from satisfying the global demand Dem_t^r while balancing resources:

$$\sum_{i \in I} Qout_{i,t}^r + Unload_t^r + P_t^r = Dem_t^r + Load_t^r + E_t^r + \sum_{i \in I} Qin_{i,t}^r \quad \forall r \in R, \forall t \in T, \quad (4)$$

with the amounts $Qout_{i,t}^r$ produced, $Unload_t^r$ withdrawn, and $Load_t^r$ filled into storage at time step t . Physical production capacities of the activated power plants are considered demanding:

$$Qout_{i,t}^r \leq z_{i,t} Cap_{i,t}^r \quad \forall r \in R, \forall i \in I, \forall t \in T, \quad (5)$$

where $Cap_{i,t}^r$ is the physical capacity in terms of the amount of resource r produced at power plant i and time step t . The conversion of resources into one another at the power plants leads to:

$$Qin_{i,t}^{r_1 r_2} = \phi_i^{r_1 r_2} (Qout_{i,t}^{r_2}, z_{i,t}) \quad \forall r_1, r_2 \in R, \forall i \in I, \forall t \in T, \quad (6)$$

where ϕ_i represents the piece-wise linear estimation of the conversion rates of the power plant i . A variable $s_{t,i}$ denotes when a power plant i goes from inactive to active at time step t . This is called activation or start-up and can be expressed mathematically as follows:

$$s_{t,i} = 1 \Leftrightarrow z_{t-1,i} = 0 \wedge z_{t,i} = 1 \quad \forall t \in T \quad \forall i \in I^{activation}, \quad (7)$$

$$s_{t,i} \leq z_{t,i}, s_{t,i} \leq 1 - z_{t-1,i}, s_{t,i} \geq (1 - z_{t-1,i}) + z_{t,i} - 1 \quad \forall t \in T, \forall i \in I^{activation}, \quad (8)$$

with the subset of power plants with activation constraints $I^{activation} \in I$. Activation of certain power plants can lead to additional costs $c_i^{activation}$ and fuel consumption $a_{t,i}^{fuelrequirement}$. The balance of fuel is given by:

$$x_{t,i}^{fuelin} = x_{t,i}^{fuelusable} + a_{t,i}^{fuelrequirement} s_{t,i} \quad \forall t \in T, \forall i \in I^{activation}, \quad (9)$$

where the total incoming fuel is denoted by $x_{t,i}^{fuelin}$ and $x_{t,i}^{fuelusable}$ represents the fuel that is left to generate heat and power.

Some power plants cannot be switched on and off immediately but require a minimum number of time steps to be activated before a start-up or deactivated before a shutdown, modelled by minimum up and down time constraints:

$$\sum_{\tau \in T_{t,i}^{up}} z_{t,i} \leq 0, \sum_{\tau \in T_{t,i}^{down}} (s_{t,i} + z_{t,i} - 1) \leq 0 \quad \forall t \in T, \forall i \in I^{activation}, \quad (10)$$

where $T_{t,i}^{up/down}$ represents the set of time steps for which a power plant i must be on (or off) after time step t .

Certain power plants $I^{ramp} \in I$ cannot change their power output immediately but must ramp up to a given level. To model ramping, we imply a limit on the amount of change of the power output of plant i from t to $t + 1$:

$$x_{t+1,i}^{out} - x_{t,i}^{out} \leq a_i^{rampup}, x_{t,i}^{out} - x_{t+1,i}^{out} \leq a_i^{rampdown} \quad \forall t \in T, \forall i \in I^{ramp}. \quad (11)$$

Finally, storage management involves

$$Storage_{t+1}^r = Storage_t^r + \delta_t Load_t^r - \delta_t Unload_t^r \quad \forall r \in R, \forall t \in T, \quad (12)$$

where $Storage_t^r$ is the stored amount of a resource. Furthermore, storage loading is subject to storage limits $Storage_{lim}_t^r$, filling limits $Load_{lim}_t^r$, and withdrawing limits $Unload_{lim}_t^r$:

$$Storage_t^r \leq Storage_{lim}_t^r \quad \forall r \in R, \forall t \in T, \quad (13)$$

$$Load_t^r \leq Load_{lim}_t^r \quad \forall r \in R, \forall t \in T, \quad (14)$$

$$Unload_t^r \leq Unload_{lim}_t^r \quad \forall r \in R, \forall t \in T. \quad (15)$$

We assume empty storage at the start of the optimization period:

$$Storage_0^r = 0 \quad \forall r \in R. \quad (16)$$

For more information on the model, additional generation technologies beyond those used in this study, investment planning extensions, and a discussion of the problem class, please refer to Clarner *et al.* (2022).

3 A-POSTERIORI ALGORITHMS FOR MULTI-OBJECTIVE OPTIMIZATION

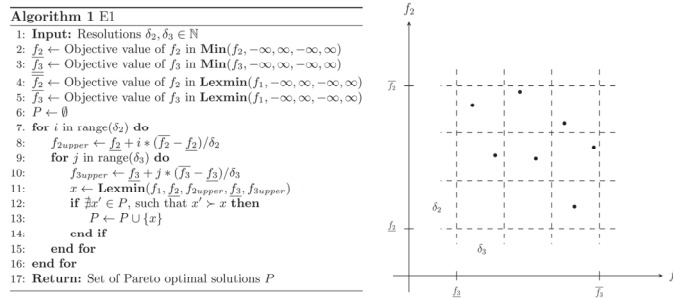


Figure 2: E1 algorithm and illustration of the grid for $n = 3$.

There are three distinct approaches to multi-objective optimization, which differ according to when the decision maker inputs their preferences. A-priori methods are algorithms in which decision-makers assert their prioritization of objective functions at the outset of the process. Conversely, in a-posteriori methods preferences are stated at the conclusion. Interactive methods have dialogue phases with decision-makers to refine prioritization iteratively throughout the process. To ensure the diversification of solutions and evaluate the pros and cons from a decision maker’s perspective while mitigating the influence of personal bias on the resultant solutions, we opted to implement two distinct a-posteriori algorithms. The main idea of both algorithms consists in partitioning the feasible space into an $n - 1$ -dimensional grid along the f_2, \dots, f_n hyperplanes.

3.1 Algorithm E1

The ϵ -constraint algorithm is one of the most commonly used a-posteriori algorithms for multi-objective optimization (e.g., Mavrotas *et al.*, 2009). It evenly divides the objective space along each dimension. Figure 2 describes the algorithm for $n = 3$ objective functions. For each dimension i in $[2, \dots, n]$ and corresponding objective function f_i , the lower bound \underline{f}_i denotes the minimum value with respect to f_i and the upper bound \overline{f}_i the lexicographic minimum with respect to f_i , giving the highest priority to f_1 . For a fixed resolution vector δ , each interval is divided into δ_i evenly spaced subintervals. The algorithm then searches every grid cell for an optimal solution.

This approach is designed to find diverse Pareto optimal solutions in a large area, providing a comprehensive depiction of the Pareto front. Nevertheless, there are scenarios where our focus may be directed towards a specific localized region harboring numerous solutions, while the remainder of the solution space may be infeasible. Algorithm E1 fails to handle these cases, leading to the development of alternative algorithmic strategies.

3.2 Algorithms E2A and E2B

We propose two algorithms highly inspired by Laumanns *et al.* (2006) adapted to characteristics of the multi-energy unit commitment. In contrast to Algorithm E1, the grid will be created dynamically at runtime, enabling a refined division of the feasible space to retrieve a high resolution in densely populated areas. Algorithms E2A and E2B employ distinct criteria for partitioning grid cells.

For algorithms E2A and E2B, additional variables G_2 and G_3 will store the grid structure by saving the ticks along the dimensions f_2 and f_3 . A list Q stores infeasible or dominated regions. Starting with a single cell grid $\left(\left[\underline{f}_2, \overline{f}_2 \right] \times \left[\underline{f}_3, \overline{f}_3 \right] \right)$ ticks are added to G_2 and G_3 based on solutions retrieved to refine the grid further.

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Algorithm 1 E2A
1: Input: Resolutions  $\delta_2, \delta_3 \in \mathbb{R}$ 
2:  $f_2 \leftarrow$  Objective value of  $f_2$  in  $\text{Min}(f_2, -\infty, \infty, -\infty, \infty)$ 
3:  $\bar{f}_2 \leftarrow$  Objective value of  $f_2$  in  $\text{Min}(f_2, -\infty, \infty, -\infty, \infty)$ 
4:  $\underline{f}_2 \leftarrow$  Objective value of  $f_2$  in  $\text{Lexmin}(f_2, -\infty, \infty, -\infty, \infty)$ 
5:  $\bar{f}_3 \leftarrow$  Objective value of  $f_3$  in  $\text{Min}(f_3, -\infty, \infty, -\infty, \infty)$ 
6:  $\underline{f}_3 \leftarrow$  Objective value of  $f_3$  in  $\text{Lexmin}(f_3, -\infty, \infty, -\infty, \infty)$ 
7:  $Q \leftarrow \emptyset$ 
8:  $G2 \leftarrow \lfloor \frac{f_2}{\bar{f}_2} \rfloor$ 
9:  $G3 \leftarrow \lfloor \frac{f_3}{\bar{f}_3} \rfloor$ 
10:  $g \leftarrow 1$ 
11:  $i \leftarrow 0$ 
12: while  $i < g^2$  do
13:    $r \leftarrow \lfloor i/g \rfloor$ 
14:    $c \leftarrow i_g$ 
15:    $q \leftarrow [G2[c], G2[c+1], G3[r], G3[r+1]]$ 
17:   if  $q \in Q$  then
18:      $i \leftarrow i + 1$ 
19:   continue
20:   end if
21:    $x \leftarrow \text{Lexmin}(f, q[0], q[1], q[2], q[3])$ 
23:   if  $x$  is infeasible then
24:      $Q \leftarrow Q \cup q$ 
25:    $i \leftarrow i + 1$ 
26:   Continue
27: end if
28:   if  $\exists x' \in P$ , such that  $x' \prec x$  then
29:      $i \leftarrow i + 1$ 
30:     Continue
31:   end if
32:    $P \leftarrow P \cup \{x\}$ 
33:    $f_{2,\text{mid}} \leftarrow (G2[c] + G2[c+1])/2$ 
34:    $f_{3,\text{mid}} \leftarrow (G3[r] + G3[r+1])/2$ 
35:   if  $(f_{2,\text{mid}} - G2[c]) \geq \delta_2$  and  $(f_{3,\text{mid}} - G3[r]) \geq \delta_3$  then
36:      $G2.\text{insert}(c, f_{2,\text{mid}})$ 
37:      $G3.\text{insert}(r, f_{3,\text{mid}})$ 
38:      $g \leftarrow g + 1$ 
39:      $i \leftarrow 0$ 
40:   end if
41:    $i \leftarrow i + 1$ 
42: end while
43: Return: Set of Pareto optimal solutions  $P$ 

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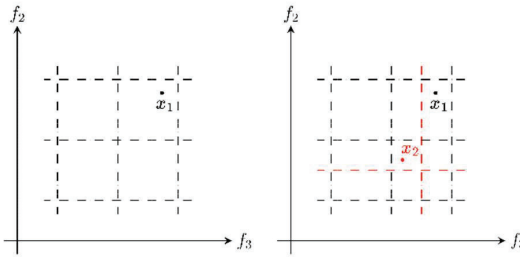


Figure 3: Algorithm E2A and illustration of dynamic grid splitting

In a row wise fashion, the algorithm computes for an unvisited grid cell q the lexicographically optimal solution. Once a feasible, non-dominated solution is found, the dimensions f_2 and f_3 is partitioned at the respective midpoint of the current cell, as long as this does not create intervals smaller than the minimum defined size. The cell tracker i is reset, g is increased, and the cell iteration process is restarted (lines 34 - 45). When the process iterates through the grid without creating new intervals, it is stopped and returns the list of Pareto optimal solutions P .

Algorithm E2B differs from E2A in the cell split condition. Instead of using resolutions δ_2 and δ_3 to store the minimum cell-width, E2B only utilizes a single parameter s , which refers to the maximum number of splits. After this number of splits has been reached, the algorithm searches the grid one last time for solutions without creating new cells. The rationale for this methodology lies in the observation that employing E2A in a densely populated feasible space results in behavior akin to that of the E1 algorithm. Consequently, this may lead to prolonged computational time, particularly noticeable for smaller values δ_2 and δ_3 . To mitigate this, constraining the frequency of grid cell splits allows for managing overall runtime without sacrificing the ability to obtain finer differentiations in targeted regions.

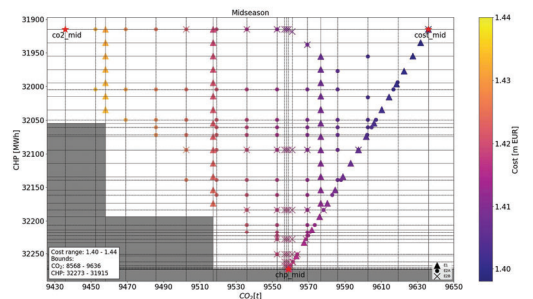
As previously indicated, we did not implement the original version of Laumanns' algorithm, which is specifically designed for optimization scenarios characterized by a finite number of solutions. Thus, after identifying a solution, the coordinates of f_2 and f_3 are utilized as new splitting points for the field, eschewing the use of midpoints (lines 38-45). Likewise, the original algorithm lacks consideration for the input resolutions δ_2 and δ_3 . Consequently, no evaluation is made regarding the potential size of the new subintervals (line 38). The optimization problem concerning Berlin's district heating network exhibits a dense feasible space. When partitioning the grid at the precise coordinates of identified solutions, there exists a significant likelihood that each of the resulting four grid cells contains similar feasible points. This leads to a pattern where the algorithm fixates on a small segment of the feasible region, a scenario avoided by the aforementioned modifications.

4 COMPUTATIONAL RESULTS FOR TWO CASE STUDIES

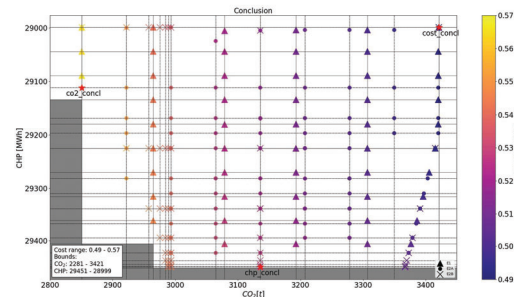
To assess the performance of the algorithms and how they can assist the analysis of district heating system operators, we showcase two case studies: A first study considers a medium sized city, which features all important heat production units but is efficient to solve, allowing us to explore the strength of the different algorithms in estimating the Pareto front. A second study involves the application of algorithm E1 to the entire district heating system of Berlin, Germany.

4.1 Case Study 1: A medium sized city

For Case Study 1, all calculations were performed using a single Intel(R) Xeon(R) Gold 6342 CPU. The memory was limited to 10GB, and the time to 50 hours. All calculations were completed well below the specified time limit. The considered model represents a realistic sub-network of the Berlin district heating system, comprising approximately 17,000 connected households. The purpose was to employ multi-criteria unit commitment optimization to analyze operational strategies within district heating networks. Within this sub-network, characterized by a centralized structure, three power plants were considered. The primary power plant contributed 98.4% of the overall heating capacity. It features two technologies, one utilizing gas (81.6% of the total capacity) and one representing a combined heat and power plant (CHP) using biomass (16.8% of the total capacity). Two additional CHPs constituted 1.3% (CHP1), using gas, and 0.2% (CHP2), using biogas, of the total capacity. A small storage facility was incorporated, and the potential for importing heat was explored, though not utilized in the algorithmic solutions. The resulting mathematical model is based on a graph with 128 nodes and 171 edges. A time step of 4 hours was considered, resulting in a constraint matrix with 68-70 T rows and 71-74 T columns per considered month. A MIP Gap of 0.1% was tolerated.



a) Midseason



b) Conclusion

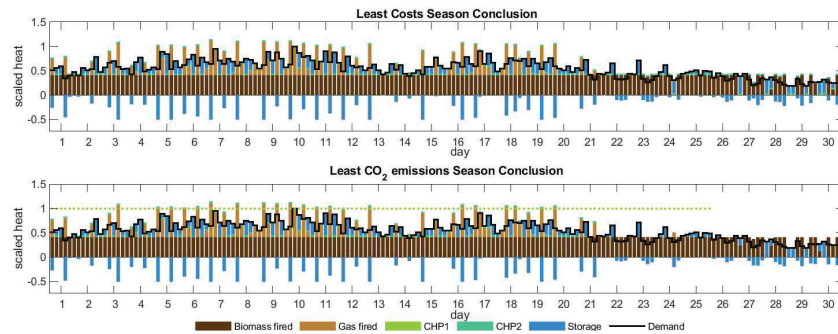
Figure 4: Case Study 1: Solutions on the Pareto front for two months representing midseason and conclusion of the heating season derived by the three algorithms (triangles for E1 and circles for E2A and x for E2B), grid structure and detected infeasible regions (grey background). For each month, three solutions are highlighted, representing the extreme points with respect to the three target functions in the selected area of the feasible space.

Figure 4 illustrates solutions on the Pareto front with respect to the three target functions for a month during the midseason and a month during the conclusion of the heating season. Overall, the three objectives are conflicting, as demonstrated by the cost-optimal solution, which involves the lowest efficiency and highest CO₂-emissions. High CHP heat production and low emissions combinations are infeasible due to the fuels used in the CHP plants. However, for both months, we find specific sweet spots where operations can reduce CO₂-emissions at a more than linear rate by decreasing CHP heat. This analysis would not be possible using a simple scaling of the three objectives.

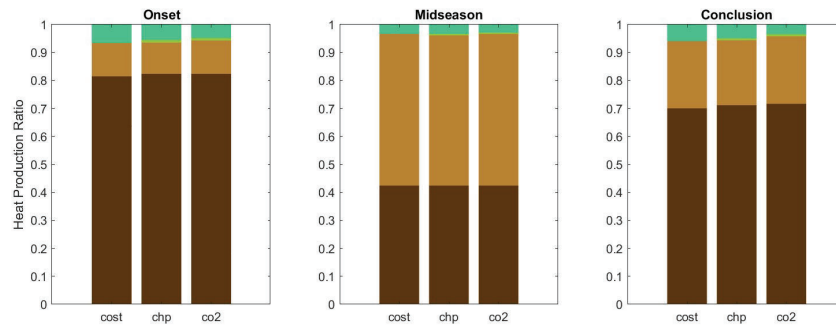
While E1 creates a regular grid, the two dynamically generated grids by E2A and E2B delve deeper in interesting areas of the solution space. Despite the mixed-integer nature of the problem, the feasible space is densely covered. Therefore, selecting the most important region of the solution space and generating the most suitable search grid are pivotal when choosing the algorithmic strategy. E1, generating a grid in the beginning, allows for effective parallelization. On the other hand, the E2B algorithm is preferable for steering the solution process to interesting regions and finding superior solutions with respect to the second and third objectives. In operational analysis, the decision maker can prioritize the second or third objective, specifically focusing on altering the grid traversal from rows to columns or vice versa. While the majority of grid boxes exhibit solutions situated in the upper right-hand corner, signifying elevated CO₂-emissions and minimal CHP heat in this region, certain boxes do not present any solutions. These unpopulated boxes denote regions that are either infeasible (depicted in grey) or dominated points. For instance, in Figure 4b, the solution at 9576 t of CO₂-emissions and 31955 MW CHP-heat surpasses solutions with identical costs and emissions but with less CHP heat, indicating inferior efficiency with respect to the third target. Similarly, in the lower right-hand corner, multiple boxes lack solutions, indicating that lower CO₂-emissions can be attained without an increase in costs or a decrease in CHP-heat.

Figure 4 displays solutions in the feasible region close to the cost-optimal point. The relative flexibility is computed from relaxing bounds on one of the targets. For example, when optimizing for CO₂ without imposing any bounds on CHP-heat, emissions can be reduced to 8568 t for the midseason month and 2281 t for the conclusion month. Achieving these reductions, however, comes at a significant cost increase and a decrease in CHP-heat. Flexibility in terms of costs and CO₂ is lower for the midseason month, with all solutions only differing 2% w.r.t. costs, 11% w.r.t. CO₂-emissions, compared to the conclusion month, displaying differences of 15% w.r.t cost and 33% w.r.t. CO₂-emissions. However, in the following analysis, we constrain CO₂-emissions even more, adhering to values in line with the bounds defined by CHP-heat.

To delve deeper into the qualitative differences between the solutions, we select the most extreme solutions in terms of each of the objectives cost, co2, and chp within the search space for the three months considered onset, midseason, and concl of the heating season. Figure 5 illustrates the time series of two of these solutions along the Pareto front, emphasizing minimal CO₂-emissions co2_concl and the cost-optimal solution cost_concl in the conclusion month. It showcases a strong preference for biomass over gas in the primary plant and distinct variations in the utilization of the CHP plants across different solutions, a characteristic also displayed in the onset and midseason period. For instance, in the cost_concl scenario, CHP1 remained unused, while in the co2_concl scenario, it contributed to the production in a majority of the time steps. As the heat demand decreased in the final days, the full capacity of the biomass plant was no longer required, and the storage facility distributed the generated heat throughout the day. Notably, CHP2 was favored in the cost_concl scenario, accounting for 6% of total energy production, compared to 4% in the co2_concl scenario (see Figure 5b), related to the different fuels and their implied CO₂-emissions. This figure also presents the total production ratios of the production units, highlighting limited alternatives for heat production within the current network setup, with differences primarily observed in the production rates of the small CHP plants.



a) Heat demand and supply time series for heating season conclusion, scaled by maximum demand. Due to limited capacity, activation of CHP1 is also depicted by a green star.



b) Production ratios for three selected solutions in each month representing minimizing costs, maximizing CHP-heat and minimizing CO₂-emissions

Figure 5: Case Study 1: Comparison of heat supply for three representative months and selected solutions from the Pareto front

4.2 Case Study 2: The full Berlin model

Case Study 2 is designed to examine the multi-objective unit commitment of the most complex Western European district heating grid for an entire year. Based on the results of Case Study 1, algorithm E1 was selected due to the option to run in parallel and efficiency without unnecessarily high resolution in densely populated areas of the solution space. The full Berlin model includes powerplants at 10 generating substations in the so-called Verbundnetz, the main connected grid, excluding the subgrid considered in Case Study 1. The heat producing power plants comprise approximately 43% installed heat capacity by heating stations and 57% by CHP plants. Most of them are primarily operated by gas (75%) and coal (19%). Furthermore, the model contains several storage facilities as well as potential import and export of heat.

The resulting mathematical model is based on a graph with 2.9 T nodes and 3.8 T edges. We selected a time step of 24 hours, resulting in a constraint matrix with 2.9 M rows and 3.0 M columns, aimed at optimizing three objectives. Due to the increased size of the model, the memory has been increased to 100GB, sufficient for the majority of the instances. A MIP Gap of 0.5% was tolerated. Only two sub-instances had to be re-executed with a memory limit of 300GB. Moreover, to achieve tractability within a reasonable time, for these specific instances the MIP-Gap was increased to 1% and 3%, respectively.

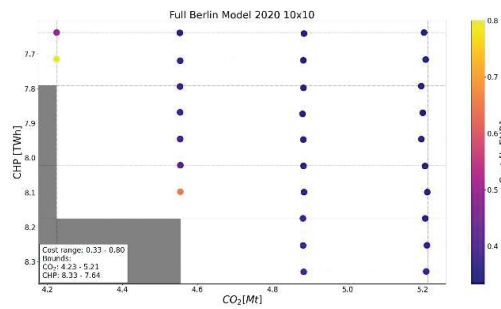


Figure 6: Case Study 2: Solutions for the full Berlin Model computed by the E1 algorithm

Given the more diverse production unit portfolio, the network's flexibility is greater compared to the outcomes of Case Study 1. The exploration range of E1 spans from approximately 4.2 Mt to 5.2 Mt of CO₂-emissions and 7.6 TWh to 8.3 TWh of CHP-heat. Within this range, we identified 29 non-dominated solutions on the Pareto front (see Figure 6). Among them, the two closest to the infeasible region, featuring low CO₂-emissions and high CHP-heat production, incur cost increases of 2.4 and 1.9 times the costs of the cost-optimal solution. Note that these are the two instances that also involved more memory and further relaxation of MIP bounds to retrieve a solution w.r.t the three objectives. In contrast, 21 of the computed solutions exhibit reduced emissions or increased CHP potential, with cost increases of less than 10% compared to the cost-optimal solution.

Given these findings, it is recommended that, upon implementation in industrial settings, the system operator adjusts the cost constraints to ensure they fall within an economically viable range, thereby mitigating the need for excessive computational resources, as demonstrated for the two grid boxes proximal to the infeasible region.

5 CONCLUSION

In summary, our study introduces a model for optimizing operations of realistic district heating networks with multiple objectives. We explore two algorithmic strategies to efficiently estimate the informative region of the Pareto front, showcasing their effectiveness in two case studies ranging from a medium sized city to the capital of Germany. The E1 algorithm offers a broad overview of flexibility options, allowing for parallelization and reduced computation time in large-scale systems with millions of constraints. These features make it preferable to explore options using very large and complex models, such as those used for day-ahead planning or longer planning horizons for district heating systems and other multi-energy models. They are also useful for other applications of unit commitment and investment models with a densely populated feasible space. Conversely, the E2B algorithm excels in steering the solution process within dense solution clusters, making it particularly useful when focused on specific regions of the Pareto front, which can especially support intra-day operations of sub-systems or small networks. Finally, combining both algorithms in an interactive workflow enhances the effective estimation of the Pareto front for holistic planning.

Our case studies reveal that the current centralized system has limited flexibility in curbing CO₂-emissions or improving efficiency to meet demand. The focus on realistic networks and their current energy production portfolios limits the use of renewable technologies to biomass in heat plants or CHP plants. In these networks, flexibility options are most prevalent at the beginning and end of the heating season, when the system operates well below maximum capacity. As we transition towards decentralized heat production to achieve decarbonization, significant changes are expected. This includes a more diverse and decentralized production portfolio, which will enhance operational flexibility and necessitate decision support systems to inform stakeholders about the various available options. Finally, multi-objective optimization emerges as a crucial tool not only for unit commitment decisions but also for guiding investment choices during this transition. Clarner *et al.* (2022) have extended this model to incorporate investment decisions, albeit at the cost of increased constraint

complexity and additional coupling constraints. This expansion further underscores the significance of the proposed approach in shaping an evolving landscape of district heating networks.

NOMENCLATURE

C	price of a resource or operational costs	Q_{in}, Q_{out}	total amount of a resource flowing into/out of a power plant
P	purchased amount of a resource	$Load$	amount of a resource loaded into storage (and its limit)
S	selling price per unit of a resource	$Unload$	amount of a resource unloaded from storage (and its limit)
E	excess amount of the produced resource	$Storage$	storage of a resource (and its limit)
Cap	physical production capacity at a power plant	MIP	mixed integer program
Dem	global demand	δ_t	length of time step t .
f	CO ₂ emissions factor at a power plant	ω	a piecewise linear function that depicts variable costs
x	fuel	ϕ	a piecewise linear function that depicts the rate of conversion of one resource to another at a power plant
a	activation variables of a power plant, e.g., representing activation costs or ramp up limits		
z	operation variable of a power plant		
s	activation switch of a power plant		

Subscripts and Superscripts

$t \in T$	time step
$i \in I$	energy conversion technology, alternatively power plant, e.g., CHP, heating station etc.
$r \in R$	resources on the network, e.g., heat, fuel, power
$activation$	indication of the subset of power plants with activation constraints

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