A thermodynamic non-equilibrium model for the expansion of a real gas in a turbine cascade

Enrico Sciubba	**Federico Zullo**
Department of Mechanical & Industrial Engineering Niccolò Cusano University, Roma, Italy *enrico.sciubba@fondazione.uniroma1.it*	Department of Civil Engineering, Architecture, Land, Environment and Mathematics University of Brescia, Italy & INFN, Milano Bicocca, Piazza della Scienza 3, 20126, Milano, Italy *federico.zullo@unibs.it*

ABSTRACT

The paper addresses the problem of the evolution of systems from a given initial state to a final one in the most general case in which the transformations include non-equilibrium states. The specific phenomenon addressed here is the expansion of a real gas (non-constant specific heats) in a turbine cascade (nozzle + rotor). The model is based on an equation of motion similar to the real Ginzburg-Landau equation but rephrased in terms of exergy, and it was described in previous publications. The fundamental assumption here is that the evolution of the fluid is driven by the specified temperature and pressure gaps between the up- and downstream boundaries, but that the details of the intermediate states are linked to the local deflection angle β. Thus, for a fixed initial pair (T_0, p_0) and an assigned degree of reaction, it is possible to explicitly express the local work, friction- and heat losses along the passage as functions of the (integral of the) head coefficient ψ. This paper may be considered as a corollary to the proof of the existence and quantification of a non-equilibrium exergy presented in previous articles by the present Authors. The model does not make use of the local equilibrium assumption, and for simplicity's sake in the example discussed here a quasi-1D approach is adopted, assuming that at each station along its path the fluid is homogeneous in the directions perpendicular to the main motion (radial velocity identically zero and tangential velocity constant in the circumferential direction).

The model calculates the (transversally averaged) non-equilibrium exergy at each station along the chord, and the main result is that its value at rotor exit is substantially higher than its equilibrium counterpart. The evolution history depends strongly on the deflection angle, i.e. on both the head and flow coefficients, ψ and ξ respectively. The solution to the mass- and energy balances leads to an analytical expression for the non-equilibrium exergy. The paradigm is theoretically simple and the resulting model of relative ease of implementation (the solution presented here was obtained on a I5 core using MATHEMATICA), and fully two-dimensional solutions may be obtained as well, provided a proper form of the heat equation is used to calculate the fluid-to-wall thermal diffusion. Applications of the proposed framework may help designers to gain a better insight into real non-equilibrium expansion processes and to more accurately tune the nozzle- and rotor efficiency.

KEYWORDS: non-equilibrium thermodynamics; exergy; Ginzburg-Landau equation; gas turbine cascade, real cascade exergy effectiveness

LIST OF SYMBOLS

Entity and units	**Symbol**	**Entity and units**	**Symbol**
Womersley number	a_W	Non-Equilibrium System	*NES*
Angular frequency, s^{-1}	ω	Pressure, Pa	p
Area	A	Radius, m	r

Area ratio	α	Height, m	z
Available Energy, J (eqtn. 3)	A	Local Equilibrium Assumption	*LEA*
Density, kg/m^3	ρ	Local expansion ratio	π
Dimensionless coordinate	Y	Mass, kg	M
Dimensionless pressure ratio	π	Polytropic exponent, $\eta_{pol}(\kappa-1)/\kappa$	γ
Dimensionless temperature	θ	Specific heat, J/(kgK)	c
Dimensionless velocity	$\mathcal{V}$	Specific heat ratio c_p/c_v	κ
Drag coefficient along the blade	ζ	Temperature, K	T
Dynamic viscosity, kg/(ms)	μ	Temperature scaling factor, T_1/T_0	θ
Entropy, J/(kgK), J/K	s,S	Time, s	t
Exergy, J/kg, J	e, E	Velocity, m/s	V
Flow coefficient	ξ	Volume, m^3	V
Head coefficient	ψ	Work, J	W

1 – INTRODUCTION

A nonequilibrium system (NES in the following) can be described in simple but accurate terms as one that whose point in the state space is evolving in time and whose evolution is driven (also) by internal gradients of the fundamental state parameters. A system relaxing to equilibrium is also a NES, because its internal properties are changing in time, and so are the internal fluxes of mass- or energy. Another, very useful for tutorial purposes, type of NES is one that is in contact with two -or more- constant temperature/mass reservoirs at different but constant temperatures/concentrations: in this case non-equilibrium is maintained by external energy or mass transfer through the boundaries and the system is said to be in steady-state non-equilibrium (SS-NES). A non-equilibrium state thus implies dynamic (i.e., time-dependent) phenomena and the presence of currents of the conserved quantities from one part of the system to another.

There is a tendency in the engineering literature to consider the lifetime of a non-equilibrium state as "very short", in the sense that its time constants are substantially lower than those of the "equilibrium situation": this is though a fallacy that can be easily dispelled by considering for instance the dynamics of turbulent flows (which are essentially in non-equilibrium for their entire life!). In this regard, a SS-NES constitutes a particularly interesting situation, because (except for the smaller molecular scales) it displays no time dependence at any point within its boundaries, but mass- and energy flow through it, driven by injection in one portion of the boundary and subtraction on another portion. Since the dynamics of real systems are dissipative, the steady state is associated with energy dispersion/degradation, i.e. with a (in this case constant) rate of entropy generation or, in exergy terms, of exergy destruction.

It is common to distinguish [3,5,15] between non-equilibrium states and non-equilibrium processes: it is clear though that the latter is a succession of several instantiations of the former, and therefore we shall consider this distinction irrelevant for the purpose of this study.

A very convenient way to treat NES is the so-called *local equilibrium assumption* (LEA), which posits the existence within the "macroscopic" NES of a large (in some norm) number K of sufficiently small regions ΔV_k in which the thermodynamic properties are related to the state variables by the very same equations as in equilibrium. The LEA has been successfully applied to many physical processes in which local gradient are smooth and diffusion times are substantially shorter than the externally imposed gradients: it does not apply to shock waves, explosions or rarefied gas flows (high Knudsen number). LEA allows for the extension of equilibrium thermodynamics to non-equilibrium systems and leads to the calculation of mass- or volume-averaged temperature, energy, entropy and exergy that may vary both in space and in time [15]. If needed, the NES global properties can then be obtained by integrating the relevant quantity (called the *density* of the variable under consideration) over the system's mass- or volume. Notice that, though each small portion ΔV_k of the NES undergoes an irreversible transformation, its entropy generation rate is not necessarily positive, because it depends on the internal material and energy exchanges: nonetheless, Second Law must obviously apply to the body as a whole. Though often criticized by theoreticians, LEA is an extremely useful and successful method:

to witness, consider that all the numerical structural-, thermal- and fluiddynamic codes are implicitly based on it.

The non-equilibrium thermodynamic description we are after stems from three observations that emerge from the literature dedicated to this branch of thermodynamics:

a) The level of irreversibility of any intermediate NE state in a process is related to its distance from global equilibrium (GE) and usually grows with it;

b) This distance NE-GE can be measured in terms of the gradients of one or more state parameters φ_i of the system: ΔT, Δp, Δu, Δz…;

c) The higher the gradient $\Delta\varphi$, the larger is the distance NE-GE and the stronger the tendency of the system to return to equilibrium or to switch to a different configuration, closer to a (possibly different) equilibrium (this "jumping" or "morphing" characterizes cases of bifurcating and catastrophic processes, which are outside of the scope of this paper).

The need for treating non- equilibrium systems was well known to Maxwell, Boltzmann and Gibbs, and also Lotka [16] addressed it albeit in a qualitative way. The topic received enormous attention after the publication in 1944 of Erwin Schrödinger' book "What is Life?", in which he suggested that life is a manifestation (he used the suggestive term "property") of far-from equilibrium systems that maintain their organized, low-entropy state by producing entropy at the expenses of an external supply of energy and matter. Since the publication of the Onsager reciprocal relations in 1931 [18,19] (see also [1,17]), many fundamental exploratory studies in the theoretical and practical aspects of non-equilibrium thermodynamics were published: it suffices here to mention the monographs by Prigogine in 1967 [21], DeGroot and Mazur in 1969 [3], Fitts in 1962 [5] and the more recent works by Demirel [4], Jou et al. [12], Grmela et al. [10] and Öttinger [20].

The thermodynamics of irreversible processes uses transport equations for temperature, pressure, and mass (concentration): its applications include the estimate the rate of entropy production in physical, chemical, and biological processes that involve heat and fluid flux, heat and mass transport, phase separation, and chemical reactions. In a non-equilibrium system, under the LEA hypothesis, the intensive variables are well-defined locally, while the densities of extensive variables, such as energy and entropy, are defined in terms of local temperature and concentrations. The thermodynamic variables are therefore functions of position and time.

3 – THE GINZBURG-LANDAU HYPOTHESIS AND GAGGIOLI'S AVAILABLE ENERGY EVOLUTION

In the context of pattern formation, in 1950 Landau and Ginzburg [13] proposed what came to be known as the real Ginzburg-Landau equation (rGL), *a* wave a*mplitude equation* to be applied to convection in binary mixtures.

The rGL equation states that, for any thermodynamic function $u(x,t)$

$$\frac{\partial u}{\partial t} = \boldsymbol{M}(\sigma)u(x,t) \qquad (1)$$

Where the nonlinear operator **M** depends on some control parameter σ that controls the stability of the system. Then, to lowest order in σ, and after rescaling, the amplitude *a* of the oscillations about equilibrium obeys the real Ginzburg-Landau equation (rGL) [14]:

$$\frac{\partial a}{\partial t} = \frac{\partial^2 a}{\partial x^2} + \sigma a - a|a|^2 \qquad (2)$$

Gaggioli [6,7,8] proposed to rephrase eqtn. (1) in terms of Gibbs' Available Energy A(x,t):

$$\frac{\partial A}{\partial t} = -\boldsymbol{\Phi}(t)A(x,t) \qquad (3)$$

The solution of eqtn. (3) is obviously a negative exponential, that guarantees that in any real process the Available Energy of an isolated system decreases in time. The matrix **F** is process- and material dependent.

4 – THE NON-EQUILIBRIUM EXERGY METHOD

The method we propose is logically derived from the three theoretical observations listed in section 1, and its implementation into a practical paradigm requires three additional fundamental assumptions: the

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first one mathematical, the second referring to non-equilibrium exergy [2,] and the third being our adaptation of the Ginzburg-Landau equation.

I - Any <u>spontaneous</u> process is described by differential equations of first order in time;

II - In any process of equilibration, some amount of mechanical work (equal to zero only in ideal limit cases) is delivered to the environment. This is the part of work that has <u>neither</u> been used by the system to evolve <u>nor</u> dissipated in the equilibration. If we look at the evolution of the system alone, the difference between the maximum theoretical work that may be extracted from the system in its initial state (its non-equilibrium exergy at t=0) and the part of it delivered to the environment is the "energetic driver" responsible for the evolution of the system. We will indicate this function by E^1.

III - The variation in time of the thermodynamic variables φ_i, $i = 1...n$, of the system is proportional to the variation (i.e., to the gradient) of E: in any volume V of the system, the variation in time of φ_i is $\frac{\partial}{\partial t} \int \varphi_i dV$ and this must be proportional to $\delta \int E dV = \int (\delta E) dV = \int \left(\frac{\partial E}{\partial \varphi_k} \right) \delta \varphi_k$. This means that

$$\frac{\partial \varphi_i}{\partial t} = P_{i,j} \frac{\partial E}{\partial \varphi_j} \qquad (5)$$

Since by definition E decreases in the equilibration process, so that $\frac{dE}{dt} < 0$, the signs of $\frac{\partial E}{\partial \varphi_j}$ and of $\frac{\partial \varphi_i}{\partial t}$ must be opposite, so that the (possibly non-linear) matrix $P_{i,j}$ is Hermitian and negative semi-definite (all eigenvalues negative or zero).

IV - There is one equation for every thermodynamic variable φ_i. Although constitutive equations linking two or more φ_i and φ_j can be possibly used to decrease the rank of system (5), it is also possible to recur to experimentally derived correlations (an example in section 6 below is the introduction of the polytropic efficiency). Once the system is properly posed, it is indeed easily verified that, if $\varphi_n = f_{n,m}(\varphi_m), \frac{\partial \varphi_n}{\partial t} = \frac{\partial f}{\partial \varphi_m} \frac{\partial \varphi_m}{\partial t}$.

5 – PRELIMINARY COMMENTS ON THE PROPOSED MODEL

Equation (5) has the same form as the Ginzburg-Landau eqtn.(1), but its physical meaning is substantially different. Replacing the Helmholtz free energy[2] in the rGL with the non-equilibrium exergy has two consequences:

a) The irreversible entropy generation in the equilibration process is automatically accounted for;
b) One degree of freedom is eliminated, because the initial value of E at the inflow boundary is completely defined in terms of the initial $H(x,t=0)$ and $S(x,t=t_{eq})$.;
c) Eqtn. (5) guarantees that the non-equilibrium exergy decreases throughout the process: if there is only one parameter φ, (1) reduces to $\frac{d\varphi}{dt} = -K \frac{\partial E}{\partial \varphi}$, with K a positive number. It follows: $\frac{dE}{dt} = \frac{\partial E}{\partial \varphi} \frac{\partial \varphi}{\partial t} = -K \left(\frac{\partial E}{\partial \varphi} \right)^2 < 0.$

Expanding on a comment made by Öttinger [20, p.22], the formalism expressed by eqtn. (5) includes both the "reversible" portion of the equilibration process and its "irreversible" one: in this sense, it may be seen as a simplification of the GENERIC formalism. The need for introducing two separate matrices, one to account for the reversible effects and one for the irreversible ones, is avoided here by using the exergy function: the problem of correctly addressing both parts of the phenomena is solved by the function rather than by the matricial formalism.

The link between a "driver" φ_q and the non-equilibrium exergy E is explicit, since each row of the matrix **P** in eqtn. (6) can be calculated by introducing proper transport equations for the φ_j and deriving the local value $E(x,t)$. Notice that no LEA is invoked here: the local pressures, temperatures,

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[1] Actually, the function *E(t)* is exactly the non-equilibrium exergy function defined in [11,22,23,24].
[2] For an open system, it is the Gibbs free energy [9] that must be substituted by the non-equilibrium exergy.

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concentrations etc. are determined by their own transport equation, so that <u>each point in the domain is not necessarily in equilibrium conditions throughout the evolution</u>.[3] As it will be clear in the following examples of application, "equilibrium constitutive equations" are never used. The model is thus a genuine non-equilibrium paradigm and can be applied to the most general types of system evolution, provided the assumptions listed in sections 1 and 3 are valid.

6 - ADIABATIC EXPANSION OF A GAS IN A TURBINE CASCADE

Consider the expansion of a real gas, initially at p_1, T_1, that flows through an axial cascade generating mechanical work (fig. 1). The (sub- or supersonic) adiabatic expansion takes place gradually along the channel and depends on the variation of the relative angle $\beta(x)$ and of the channel area A(x) along the centreline s[4]. Boundary layer- and turbulence effects are neglected, and $p(x)$ and $T(x)$ are assumed to be uniform on the planes (in this quasi-2D model, lines) perpendicular to s at each x. At each station x along the centreline, the gas experiences a pressure drop $dp(x)$, with a corresponding $dT(x)$, and delivers an ideal work $dW_{id}=U^2 d\psi$, where U is the peripheral velocity of the cascade and $\psi= V_t/U$ the head coefficient [25].

With reference to figures 1 and 2, let us define the following quantities:
-   The local pressure ratio $\pi(x)=p(x)/p_0$
-   The local temperature ratio[5] $\theta(x)=T(x)/T_0$
-   The local area ratio $\alpha(x)=A(x)/A_1$
-   The flow coefficient $\varphi(x) = \frac{V_m(x)}{U}$
-   The head coefficient $\psi(x) = 1 + f(x)\varphi(x)$ with $f(x) = \cot[\beta(x)]$

Along the passage, the density of the gas is allowed to change, so that the corresponding meridional velocity also varies along the centreline: $V_m(x) = V_m \frac{A_1}{A(x)} \frac{\rho_1}{\rho(x)} = \frac{\varphi U(x)}{\alpha(x)\pi(x)^{1+\gamma}}$. The following assumptions are made:

i)   The variation of the relative angle $\beta(x)$ and of the "area" $A(x)$ (in this quasi 2-D example, a segment) are known, and so is the inlet area A_1;
ii) The initial conditions of the gas are assigned: p_1, T_1, R, $\kappa=c_p/c_v$
iii) The specific heats are a function of T, but a $\kappa=f(p,T)$ is adopted for simplicity;
iv) Pressure, temperature and velocity are constant across the passage at each x-station (i.e., they depend only on x and not on y).

Two different equations describe the exergy drop for the statoric and the rotoric cascade.

6.1 – Stator
From the definition of exergy (omitting for the moment the dependence on the spatial coordinate x):
$$E_s = h - T_0 s = c_p T - c_p T_0 \left[\ln\left(\frac{T}{T_0}\right) - \frac{\kappa-1}{\kappa} \ln\left(\frac{p}{p_0}\right)\right] + 0.5 * V^2(1 - \zeta\frac{T_0}{T}) \qquad [\text{kJ/kg}] \qquad (6)$$
Where ζ is the friction loss factor and thermal recovery has been included.
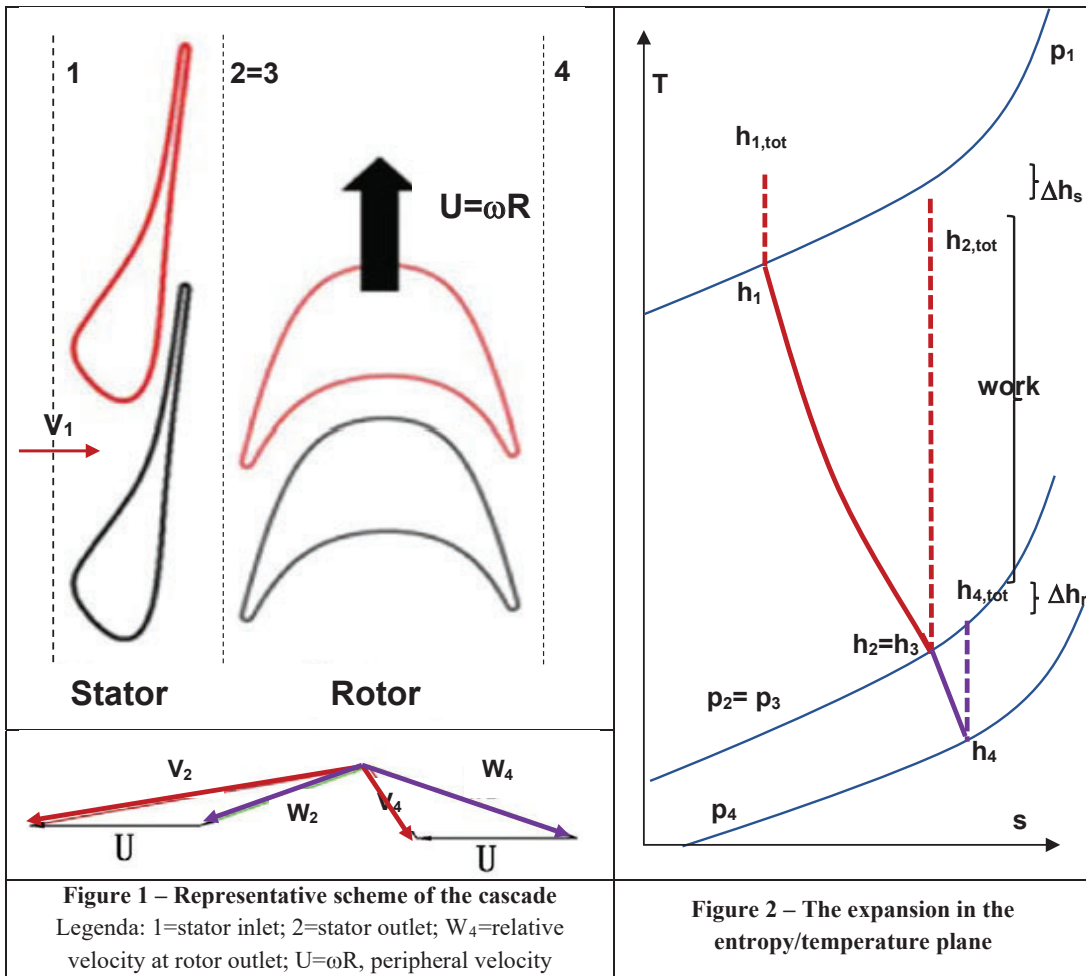Considering that $\pi = \theta^{1/\gamma}$:
$$E_s = c_p T + \varepsilon c_p T_0 \ln\left(\frac{T}{T_0}\right) + 0.5 * V^2(1 - \frac{\zeta T_0}{T}) \qquad (7)$$
with $\varepsilon = \left(\frac{1-\eta_{pol}}{\eta_{pol}}\right)$, where η_{pol} is the polytropic efficiency of the cascade

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[3] There is a limitation here: if the system contains a fluid, the model can be applied as long as the motion of the fluid is described by an analytic equation: this excludes virtually all practical fluid processes, unless a "locally averaged velocity field" is somehow specified or the relevant equations are available (creeping flows, a restricted number of simple laminar flows, etc.).
[4] We are considering here a steady-state process for which all thermodynamic quantities are *f(x)*, with *x=U(x)*t*
[5] Both θ and π are normalized to the usual reference conditions (=ambient T and p), T_0=298K and p_0=1 bar

Figure 1 – Representative scheme of the cascade
Legenda: 1=stator inlet; 2=stator outlet; W$_4$=relative velocity at rotor outlet; U=ωR, peripheral velocity

Figure 2 – The expansion in the entropy/temperature plane

In line with eqtns. (1,2), and considering that in this particular application $dx=V(x)dt$, we can assume for the evolution of T along x an exponential decay:

$$T(x) = ae^{-bx} \qquad (8)$$

Imposing the b.c.: $T(x=0)=T_1$; $T(x=\ell_s)=T_2$ we obtain $a=T_1$, $b=ln(T_1/T_2)/\ell_s$ [6].

The derivative $\frac{\partial T}{\partial x}$ takes the form:

$$\frac{\partial T}{\partial x} = -\frac{T_1 ln(T_1/T_2)e^{-bx}}{\ell_s} \qquad (9)$$

And is negative definite.

Imposing conservation of the total enthalpy we derive an equation for the velocity $V(x)$ as a function of $T(x)$:

$$V(x) = [(K_1 - K_2 T(x)]^{0.5} \qquad (10)$$

With $K_1 = V_1^2 + 2c_{p,1}T_1$ and $K_2 = 2c_{p,T}$.

And from mass conservation a correlation between T, V and A:

$$A(x) = A_1 K_3 \frac{T^\varepsilon}{V} \qquad (11)$$

With $K_3 = \frac{V_1}{T_1^\varepsilon}$.

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[6] This is equivalent to saying that the coefficients "a" and "b" are derived "experimentally"

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The temperature, velocity and area profiles are displayed in figures 3 through 6[7]. In line with the steady state assumption (see note 4) we can also use eqtn. (5) to obtain an implicit equation for the time evolution of the temperature:

$$\frac{\partial T}{\partial t} = -P\frac{\partial E}{\partial t} = [K_1 - K_2T(x)]^{0.5}\frac{T_1}{\ell_s}\ln\left(\frac{T_1}{T_2}\right)e^{-bx} ; \qquad with\ x = V(x)t \qquad (12)$$

Which, incidentally, may be interpreted as a way to calculate the coefficient P on the basis of "empirical" correlations (K_1, K_2, a, b).

For future notice, the above equation is valid in the case of a homogeneous temperature distribution (i.e., constant in y at every station x). Releasing this constraint leads to an equation of the type:

$$\frac{\partial T}{\partial t} = -P\frac{\partial E}{\partial t} + k_T\nabla^2 T = [K_1 - K_2T(x)]^{0.5}\frac{T_1}{\ell_s}\ln\left(\frac{T_1}{T_2}\right)e^{-bx} + k_T\nabla^2 T \qquad (13)$$

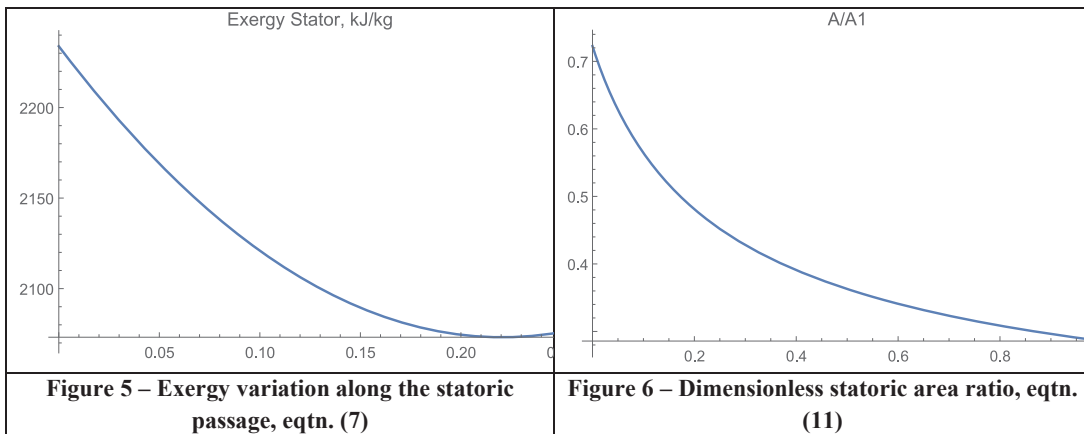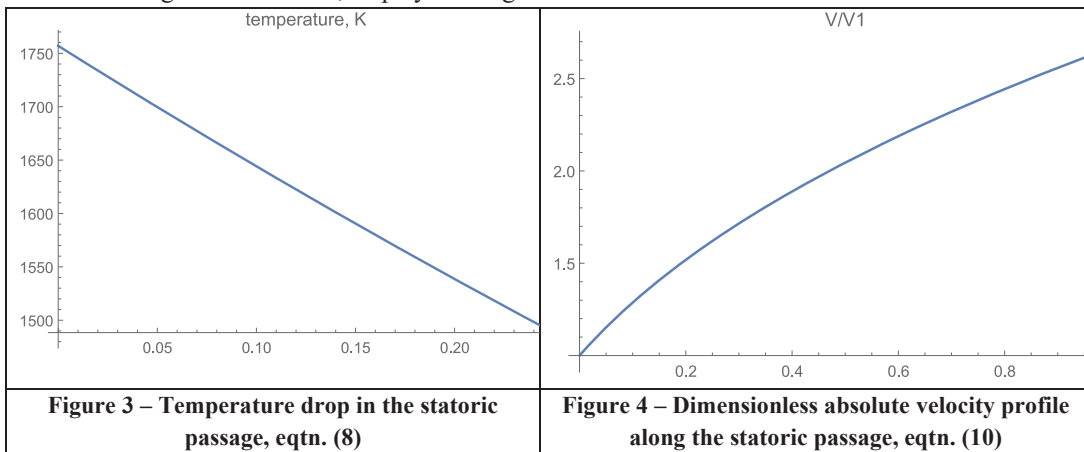That is much more complicated to solve and whose study exceeds the scope of the present paper.

We use eqtn. (10) to express the V^2 term in eqtn. (7) as a function of T(x). The derivative of the local exergy w.r.t. the local temperature is then:

$$\frac{\partial Es}{\partial T} = \frac{\partial E_S}{\partial T} = c_p + c_p\varepsilon\frac{T_0}{T} - \frac{K_2}{2} + \frac{K_1\zeta T_0}{2T^2} \qquad (14)$$

Notice that eqtn. (14) is definite positive. What we are interested in is the variation of the specific exergy along the blade. Recalling that in our 1-D model "x" denotes the coordinate along the central streamline of the blade passage:

$$\frac{dE_S}{dx} = \frac{\partial E}{\partial T}\frac{\partial T}{\partial x} = \left[\frac{\varepsilon c_p T_0^2}{T} - \frac{K_1\zeta T_0}{T^2}\right](-abe^{-bx}) \qquad (15)$$

That is the sought after relation, displayed in figure 5.

Figure 3 – Temperature drop in the statoric passage, eqtn. (8)

Figure 4 – Dimensionless absolute velocity profile along the statoric passage, eqtn. (10)

Figure 5 – Exergy variation along the statoric passage, eqtn. (7)

Figure 6 – Dimensionless statoric area ratio, eqtn. (11)

[7] A more realistic profile for *T(x)* could of course be obtained by numerical simulation or physical experiment, which would be completely in line with the idea underlying the G-L approach.

6.2 – Rotor

The ideal expansion work is:

$$w_{id} = U^2\psi(x) \ [\text{kJ/kg}]$$

Let us assume that both the head and flow coefficients vary linearly along the rotor passage:

$$\varphi(x) = \varphi_3 - (\varphi_3 - \varphi_4)x; \qquad \psi(x) = \psi_3 - (\psi_3 - \psi_4)x \qquad (16)$$

The real work includes the frictional losses and the recovery factor: mirroring the procedure adopted for the statoric calculations, the effects of friction can be introduced by considering that a portion w_f of w_{id}, is "used" to overcome viscous effects. This "lost" work is proportional via a drag coefficient ζ to the square of the local velocity (for the sake of clarity we shall omit the reference to the x-coordinate here). Since the walls are assumed to be adiabatic, w_f is transformed into an equivalent amount of heat $q_f = w_f$, a portion of which will result in an additional amount of mechanical work (this is the so-called recovery effect).

$$w_{real} = Er = w_{id} - w_f + (1 - \tfrac{T_0}{T})w_f = c_p\left(T + \varepsilon\,T_0\ln\tfrac{T}{T_0}\right) + 0.5V^2\left(1 - \tfrac{\zeta T_0}{T}\right) \qquad (17)$$

The low degree of reaction of the cascade (~0.1) allows to assume that the dimensionless area $\alpha(x)=A(x)/A_3$ be constant along the rotoric passage and equal to α_R, whose value depends on $V_{3m}=\varphi_3*U$. Using the isentropic relation we obtain:

$$T = K_4 p^n \qquad (18)$$

With $n = \tfrac{\gamma}{\gamma-1}$ and $K_4 = \tfrac{T_3}{p_3^n}$. Substituting in (17):

$$Er = K_4 c_p p^n + \varepsilon c_p T_0 \ln\left(\tfrac{K_4 V^n}{T_0}\right) + 0.5V^2\left(1 - \tfrac{\zeta T_0}{K_4 V^n}\right) \qquad (19)$$

We now want to adopt the velocity as the relevant "order parameter", $V^2(x) = V_m^2(x) + V_t^2(x)$. Using (16) and (17) we obtain an explicit expression for $E_r(x)$.

The derivatives of V w.r.t. x and of E w.r.t. V are respectively:

$$\frac{\partial V}{\partial x} = -\frac{x*(c2^2+c4^2)-c1*c2-c3*c4}{[c1^2+c3^2-2*x*(c1*c2+c3*c4)+x^2*(c2^2+c4^2)]^{0.5}} \qquad (20)$$
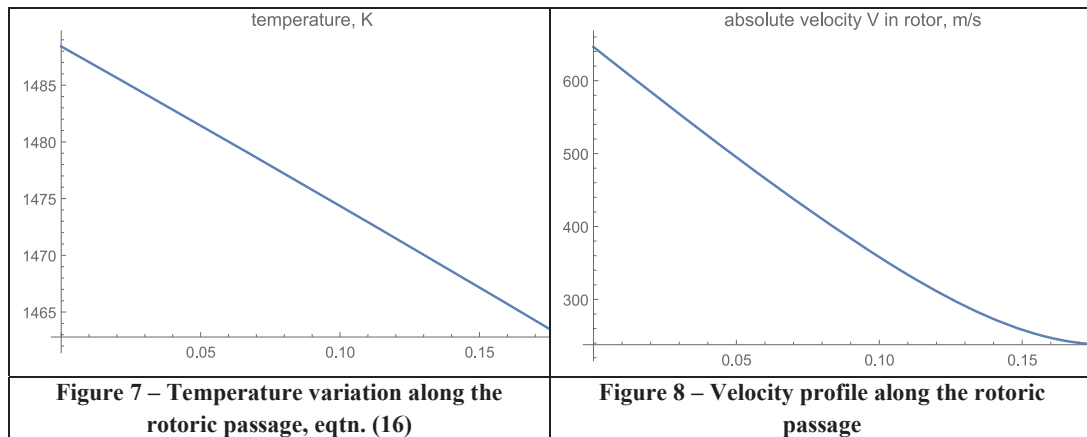
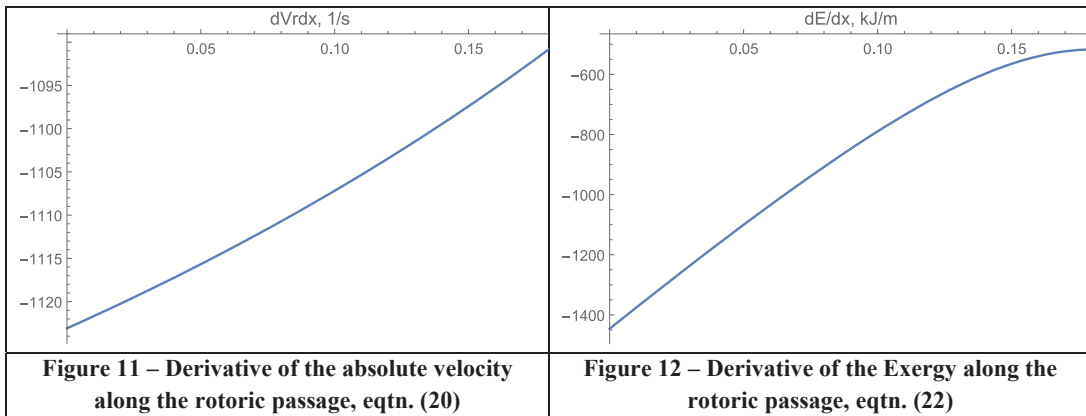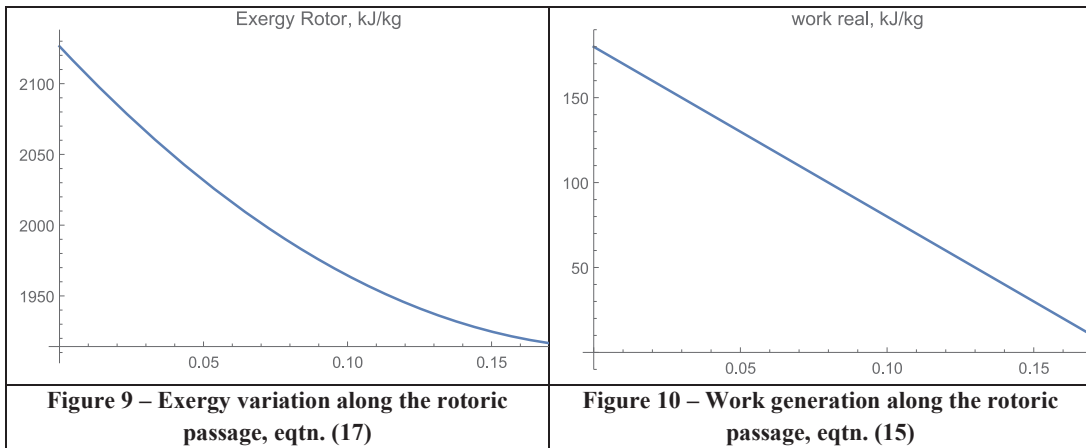With $c_1=\varphi_3$; $c_2=\varphi_3-\varphi_4$; $c_3=\psi_3$; $c_4=\psi_3-\psi_4$.

$$\frac{\partial Er}{\partial V} = 2.*V(x)[1.-\zeta*T_0/T(x)] \qquad (21)$$

Clearly, eqtn. (18) is negative definite and (19) positive (the loss coefficient ζ is of $O\langle 10^{-2}\rangle$). The variation of the exergy along the blade is:

$$\frac{dEr}{dx} = \frac{\partial Er}{\partial V}\frac{\partial V}{\partial x} \qquad (22)$$

The temperature, velocity, work and exergy profiles in the rotor are reported in figures 7 through 12.

| **Figure 7 – Temperature variation along the rotoric passage, eqtn. (16)** | **Figure 8 – Velocity profile along the rotoric passage** |

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Figure 9 – Exergy variation along the rotoric passage, eqtn. (17)

Figure 10 – Work generation along the rotoric passage, eqtn. (15)

Figure 11 – Derivative of the absolute velocity along the rotoric passage, eqtn. (20)

Figure 12 – Derivative of the Exergy along the rotoric passage, eqtn. (22)

7 – DISCUSSION

The crucial point of the procedure described in the previous sections is that it is possible to obtain an analytical expression for the exergy drop in a turbine cascade starting from general Thermodynamic principles. In fact, the geometry and the type of fluid evolution are arbitrary, and thus the model can be extended to a whatever fluid/machine interaction, provided the posited assumptions are abided by. In particular, the simplified 1-D approach taken here can be extended to a 2-D channel at the cost of some conceptual and algebraic complication.

The turbine example is interesting because it is amenable to a relatively simple demonstration of the application of the model to real cases. Assume the data reported in Table 1 refer to the cascade depicted in figure 1, with a subsonic flow throughout, a $\psi=2$, i.e. an impulse rotor with a pressure drop due almost exclusively to the frictional losses. The φ is equal to the "optimal" value reported in the literature [25]. All properties are for air, with enthalpies and specific heats calculated by MiniRefProp (except in the derivation of eqtns. 7 & 11 where an average c_p was used).

Table 1: Turbine stage 1-D design specs

U=0.5ωD, m/s	T_0, K	p_0, bar	R, J/(kgK)	κ	β	ψ	η_p	ℓ_{stator} m	ℓ_{rotor} m
300	1757	15	287	1.4	2	2.	0.85	0.25	0.18

section	V_m	V_t	V	T	p	ρ	c	Ma	E
1	240	0	240	1757	1519500	3.01	840	0.28	
2	240	600	646	1488	767424	1. 8	773	0.31	
3	240	600	646	1488	767424	1.8	773	0.31	

| 4 | 216 | 0 | 228 | 1463 | 714472 | 1.7 | 767 | 0.30 | |

8 – CONCLUSIONS

The Ginzburg-Landau real equation, besides being an interesting piece of mathematical modeling, embodies an extremely important phenomenological assumption: the evolution of a non-equilibrium system is deterministically driven by an "action". Similarly to Hamilton's principle, this intuition allows to formulate general models for the motion of the system in its state space, provided of course that: a) the correct "cause" is posited and b) that some experimental evidence connects the model to the perceived physical reality. Notice that, unlike the Hamiltonian, the "action" does not need to be a conserved quantity.

In this work, we argue that exergy is a proper descriptor of a non-equilibrium evolution is exergy and that the "order parameters" (in the G-L sense) are the relevant variables that describe the state of the evolving system. The exergy of a system is *per se* a non-equilibrium quantity (it retains a meaningful value both for equilibrium and non-equilibrium states) and is expressed in terms of the usual thermodynamic properties (pressure, temperature, velocity, electrical charge etc.). In this work, we have applied the G-L model to the simplified 1-D evolution of a gas in a realistic first stage of a modern gas turbine. The results show the variation of the exergy along the blade passage. The example is extremely simple, but it can be expanded to 2-D and possibly 3-D at the cost of a non-negligible algebraic complication: while our 1-D application neglects diffusion effects normal to the direction of motion (as a consequence of our assumption of homogeneous fluid properties along each plane perpendicular to the line of motion), in 2- and 3-D cases differential diffusion equations ought to be added both in y and z, and (∇T)^2, (∇p)^2 and the related transport terms would appear in the exergy equation.

The importance of adopting the G-L model to describe the evolution of a system resides in the possibility of adopting exergy as "the universal driver" and leads to a simplification and unification of most of the existing theories about non-equilibrium Thermodynamics.

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