Thermoeconomic Modeling as a Tool for Internalizing Carbon Credits into Thermal System Analysis

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Abstract:

According to the Intergovernmental Panel on Climate Change (IPCC), greenhouse gas emissions (GHG) have increased since 1990. Electricity and heat generation, along with transportation, accounted for more than two-thirds of emissions in 2018. As CO₂ represents the largest percentage of GHG, the term carbon has come to be adopted as a synonym for these gases in climate debates. In order to control emissions, the carbon market helps industries/sectors that are not able to meet the emission reduction goals to buy credit from the ones that have reduced their levels below the required. Thermoeconomics plays a fundamental role in the analysis of thermal systems. Therefore, this study aims to detail how thermoeconomic modeling can be used to include expenses or revenues related to the carbon market through an example in a gas turbine cogeneration system. In addition, it highlights that this modeling can be used in the internalization of other expenses such as environmental control devices, licenses, and permits. Results show that the environmental device is capable of internalizing carbon credits and systematically distributing them to the cost of final products.

Keywords:

Thermoeconomic modeling; Carbon credit; Carbon market; Environmental cost.

1. Introduction

According to the Intergovernmental Panel on Climate Change (IPCC) [1], greenhouse gas emissions (GHG) have increased since 1990. The combination of electricity and heat generation (cogeneration), along with transportation, accounted for more than two-thirds of emissions in 2018 [2]. As CO₂ represents the largest percentage of GHG, which can cause global warming, the term carbon has come to be adopted as a synonym for these gases in climate debates.

In order to control emissions, the carbon market provides industries/sectors that are not able to meet the emission reduction goals the possibility to buy credit from the ones that have reduced their levels below the required. One carbon credit corresponds, by convention, to one ton of carbon dioxide. Therefore, it can be considered an asset (financially and environmentally), representing the reduction or removal of one ton of CO_2 equivalent, which has been recognized and issued as a credit in the carbon market, regardless of whether it is voluntary or regulated [3].

This market is already regulated in some countries, such as in the European Union, which has well-defined credit values [4]. Nonetheless, in many others, such as Brazil, this market is still voluntary. Recently, the Brazilian government issued a decree [3] to regulate this market and institute the National System of Greenhouse Gas Emissions Reduction (SINARE); however, there are still no deadlines for implementation. According to the World Bank's 2022 report [5] and IPCC 2023 [6], the carbon market (associated with environmental preservation measures) is expanding worldwide, but still below the necessary levels to mitigate environmental problems and meet the environmental agenda signed in the Paris Agreement against the threats of climate change.

Thermoeconomics combines thermodynamic and economic concepts to provide pieces of information that are unavailable in conventional energetic and economic analyses. The information provided is fundamental in the design and operation of thermal systems [7]. The original objective of thermoeconomics was to mathematically combine the Second Law of Thermodynamics with economics. However, these analyses must also incorporate environmental issues [8]. In thermoeconomics, exergy is the most appropriate thermodynamic magnitude to use, because it takes into account aspects of the Second Law of Thermodynamics considering the quality of energy, locating and quantifying the irreversibilities of the

process [7]. The exergy is also the most appropriate connection between the Second Law and the environmental impact because it measures a system's state deviation in relation to the environment [9].

Multiproduct thermal system analyses such as cogeneration, in which two products (useful heat and power) are generated collectively from a single combustible, require rational criteria for distributing the cost of the combustible to the various final products. In these circumstances, thermoeconomics allows rational allocation (through physical criteria) of monetary, exergetic, and environmental costs for the final products. Therefore, it is possible to compare exergetic/monetary [10–14] and/or environmental [15–18] costs of each product with the production cost of each one in separate systems.

Thermoeconomic methodologies have already been used to include environmental aspects, such as specific CO_2 emissions. However, they were not used in the internalization of monetary costs associated with environmental issues, such as carbon credits. Therefore, the novel concept introduced in this study is to exemplify how to perform this internalization thermoeconomically.

Furthermore, it is observed that thermoeconomics has a fundamental role in energy conversion systems analyses. The main goal of this study is to detail how thermoeconomic modeling can be used as a tool to include the expenses or revenues relative to the carbon market in thermal systems analyses and allocate them to internal and final products of the system. The conventional modeling used to calculate the monetary costs of internal flows and final products can be adapted to address these environmental costs. This adaptation is detailed in matrix notation, through a case study of a gas turbine cogeneration system. It also shows how this inclusion can influence the monetary costs of the system's final products. In addition, it highlights that this modeling can also be used to internalize other costs such as environmental control devices, environmental licenses, and permits.

It is important to emphasize that the methodology used herein to exemplify and detail this internalization of environmental costs, which is the case of carbon credits, defines a device to represent the environment in the thermoeconomic diagrams and thus allocates environmental costs exactly on the environmental device. This study uses the H&S Model as the method; however, any other exergy-based thermoeconomic methodology that defines this device in a consistent way to represent the environment can be used to conduct the analysis in a similar way.

2. Thermoeconomic modeling

In addition to the conventional modeling used to determine the monetary and exergetic unit costs of the system's internal flows and final products, this section shows how the modeling is generally adapted to allocate specific pollutant emissions and further details how carbon credits can be included in thermoeconomic modeling.

2.1. Conventional Modeling

Equations (1) and (2) are used to determine the monetary (*c*) and exergetic (k^*) unit costs, respectively, of the internal flows and the systems' final products. The allocation of specific (λ) pollutant emissions, such as CO₂, NOx, and SOx can be performed through Eq. (3). In these equations, the subscripts "*out*" and "*in*" are associated with the outputs and inputs of flows, respectively. Y represents the generic thermodynamic magnitude which can be evaluated by power, heat, exergy flows, or its components. E_F is the exergy of the external combustible. c_F and k_F^* represent its monetary and exergetic unit cost, respectively. λ_F is the amount of emission generated due to the combustion of one unit of exergy from the external fuel. Z, conventionally, is the external hourly cost of the subsystem due to capital and equipment operation and maintenance.

$$\sum (c_{out}, Y_{out}) - \sum (c_{in}, Y_{in}) = Z + c_F. E_F$$
(1)

$$\sum (k_{out}^*, Y_{out}) - \sum (k_{in}^*, Y_{in}) = k_F^*. E_F$$
(2)

$$\sum (\lambda_{out}, Y_{out}) - \sum (\lambda_{in}, Y_{in}) = \lambda_F. E_F$$
(3)

Equation (2) is obtained through Eq. (1). In this case, the Z term should be zero and the exergy unit cost of the external fuel (k_F^*) is generally considered to be equal to its exergy; therefore, the exergy unit cost is equal to 1 kW/kW [7].

The monetary and exergetic unit costs can be interpreted as an economic and thermodynamic efficiency measure of a flow production process, respectively [7]. On the other hand, the balance represented by Eq. (3) can be interpreted as an environmental efficiency measure production process of this flow [16].

In all cases, Eqs. (1) - (3), auxiliary equations are generally necessary to complete the modeling equations system. These equations are defined according to the applied thermoeconomic diagram. In the case of productive diagrams, the equality criterion [19] is used. Following this criterion, all products of a subsystem have the same unit cost because they were generated in the same productive process under the same irreversibilities.

2.2. Inclusion of monetary costs of environmental charges

Equation (3) is used to allocate specific emissions to the internal flows and the thermal systems' final products, and, therefore, it is an analysis that considers environmental aspects in thermoeconomic modeling. However, it does not take into account monetary costs associated with environmental issues, such as carbon credits and environmental treatment/control equipment.

The Z term (Equation 1) is a key point in the allocation of environmental costs. In a conventional monetary cost evaluation, it represents the subsystem's external hourly rate due to the capital, operation, and maintenance. Nevertheless, it can also be used for the allocation of environmental costs through a device that represents the environment in the thermoeconomic diagrams. An energy conversion system can be defined as a set of components that interact with each other and with the environment through a set of flows of matter, work, or heat [20]; therefore, the environment is part of the system. Thus, it can be represented by an environmental device in thermoeconomic diagrams according to some models.

Equation (4) shows how a conventional thermoeconomic model of monetary unit cost (Equation 1) can be adapted to decompose the term (Z) in hourly costs due to environmental charges (Z_{env}) and capital, operation, and maintenance costs (O&M).

$$Z = Z_{cap} + Z_{0&M} + \mathbf{Z}_{env}$$

The environmental device has no acquisition cost, but it is through it that environmental charges can be internalized and redistributed to the other equipment and the final products. For instance, when installing waste control devices in a plant, such as an electrostatic precipitator for ash disposal in flue gas or a bag filter for air pollution control, one can attribute the costs associated with its capital and O&M to the environmental device or any other equipment, that has the function of mitigating environmental impacts by decreasing the amount of GHGs emitted into the atmosphere.

The same can be done for devices used in the capture and storage of carbon, environmental permits, licensing costs, fines for emitting pollutants, and any other abatement cost (cost of resources employed in the treatment or proper waste disposal). Thus, allocating the environmental charges exactly on the device in the diagram defined to represent the environment. Since the term Z is always associated with some equipment, an adequate option is to associate environmental cost with the device that represents the environment in the diagrams.

2.2.1. Inclusion of carbon credits

In addition to the monetary costs mentioned in the previous section, this paper suggests that through the environmental device, it is also possible to take into account the pricing of carbon and internalize the expenses or revenues generated by carbon credits. In this case, the Z_{env} term can be positive or negative. Negative in case of a revenue generated due to the reduction or removal of emissions (which can generate a credit to be sold) and positive in case of an additional cost of buying carbon credits by a plant that failed to meet the emission reduction targets and had to buy credits from those that reduced theirs below the stipulated levels. The full detail of the thermoeconomic modeling taking this carbon market into account is presented in section 3.

3. Case study – gas turbine cogeneration system

The thermal system chosen to exemplify how thermoeconomic modeling can be used as a tool for internalizing carbon credits is a cogeneration system with a simple gas turbine as shown in Fig. 1. This system is composed by an air compressor (AC), combustion chamber (CC), gas turbine (GT), and recovery boiler (RB). Part of the power generated by the turbine is used to drive the compressor (W_{AC}). Two final products, net power (W_N) and useful heat (Q_U), are generated from a fuel (Q_F).

The parameters of the main flows of the physical structure (obtained with the Engineering Equation Solver - EES software [21]) can be found in Table 1. Table 2 indicates the quantities of the main productive flows. The reference conditions are defined by $T_0 = 25$ °C and $P_0 = 1.0132$ bar, and under these conditions the CO₂ mass flow from the exhaust gases is $m_{CO_2} = 2228$ kg/h considering natural gas as fuel. More information of this system is available in [22]. The monetary unit cost of fuel (natural gas) is 24.04 \$/MWh, according to the average value for the year 2022 in the international market [23].

Table 3 shows the external monetary flows due to the equipment of the cycle. These values were obtained from [22] and updated through the Chemical Engineering Cost Index (CEPCI) until the year 2022 [24]. The cost of the carbon credit used was 85 \$/ton, which represents the average for the year 2022, according to [4].

The thermoeconomic modeling can be performed through different types of diagrams: physical, productive, and comprehensive. Since modeling with the physical diagram is not enough to identify the waste cost formation process [22], most methodologies use the productive diagram.

(4)

The description of the cost formation process of thermal systems based on productive flows is an original feature of the functional methodologies: Thermoeconomic Functional Analysis (TFA) [19] and Engineering Functional Analysis (EFA) [8]. However, other thermoeconomic methodologies, such as the H&S Model [22] used in this paper, have also adopted this feature.



Figure 1. The physical structure: cogeneration system.

Physical flow		m [kg/s]	TIO	D [bor]	
n. °	Description	iii [kg/5]	1[0]		
1	Air	14.72	25.00	1.0132	
2	Air	14.72	230.20	5.1040	
3	Gases	14.94	850.00	4.8480	
4	Gases	14.94	537.30	1.0207	
5	Gases	14.94	151.10	1.0132	
6	Water	2.487	60.00	20.400	
7	Steam	2.487	212.4	20.000	

Table 1. Main physical flow parameters of the system.

Equipment	Flow	Quantity [kW]
Air compressor (AC)	W _{AC}	3113.03
Combustion chamber (CC)	Q _F	11630.96
O_{res} turking (OT)	Wgt	5546.50
Gas turbine (GT)	WN	2433.47
Recovery boiler (RB)	Qu	2246.32

Table 3. Equipment external monetary cost.					
Equipment	Z [\$/h]				
Air compressor (AC)	25.33				
Combustion chamber (CC)	9.04				
Gas turbine (GT)	34.37				
Recovery boiler (RB)	21.71				

3.1. Thermoeconomic models

Thermoeconomic modeling can be carried out with the well-known E Model that uses total exergy flows to define the physical and/or productive flows of the diagrams. However, in some cases, it becomes necessary to disaggregate the exergy into components, such as to isolate dissipative equipment and carry out an adequate allocation of the waste cost in thermal systems. One such exergy disaggregation model is the H&S Model [22] which describes the behavior of thermodynamic cycles in the h-s plane considering the enthalpy

and entropy variation of the working fluid, as suggested by [25]. This is a model for disaggregating the physical exergy into its enthalpic(E^H) and entropic (E^S) parts, according to Eq. (5). The total exergy (E^{TOTAL}) can be defined by Eq. (6) as the sum of the physical (E^{PH}) and chemical (E^{CH}) components, disregarding nuclear, magnetic, electrical, surface tension, kinetic and potential effects [26].

$$E^{PH} = E^H - E^S$$

$$E^{TOTAL} = E^H - E^S + E^{CH}$$

(5) (6)

The H&S Model defines the environmental device (ENV) in the productive diagram that interacts with the other plant subsystems. In this methodology, this device plays a fundamental role in the analysis of thermal systems, especially in the treatment of waste and the internalization of environmental costs. Both the physical (represented by $E_{5:1}^H$) and chemical ($E_{3:2}^{CH}$) components of the waste are dissipated in (ENV), see Fig. 4, and this is where the system receives air from the compressor inlet. The chemical component is generated in the CC due to the combustion reaction in which the air and fuel mixture is transformed into combustion gases. The E Model does not define a device to represent the environment in the diagram.

In addition, the environmental device (used in the H&S Model) is also responsible for closing the cycle (Figure 2); thus, redistributing the waste costs to the other plant components and consequently to the final products.

Figure 2 represents the cogeneration cycle in the h-s diagram and the numbering in this diagram represents the processes performed by the following components:

- 1-2: compressor (1-2s would be isentropic compression);
- 2-3: combustion chamber;
- 3-4: gas turbine (3-4s: isentropic expansion)
- 4-5: recovery boiler.

At the exit of the recovery boiler (point 5), the exhaust gases have exergy (therefore, they are waste). Although this equipment (RB) slightly reduces the entropy of the working fluid, the cycle is not fully closed. In the case of a Rankine cycle, for example, the condenser completely closes the cycle by reducing the entropy of the turbine's output steam to that of the saturated liquid at the pump entrance.

The device representing the environment in the diagram (ENV) performs process 5-1 and completely closes the loop. In this device, flow 5 represents the exhaust gases and flow 1 is the air drawn in by the compressor.



Figure 2. The environment device in open cycles.

3.1.1. Productive diagram

Figures 3 and 4 represent productive diagrams of the gas turbine cogeneration system according to E and H&S Models. In E Model, the flows represent exergy variations between two physical states (i and j) according to Eq. (7). In the H&S Model, the productive flows represent variations of the enthalpic, entropic and chemical components of the exergy between i and j according to Eqs. (8) - (10), respectively.

$$E_{i:j} = E_i - E_j \tag{7}$$

$$E_{i:j}^n = E_i^n - E_j^n \tag{8}$$

$$E_{i:j}^S = E_i^S - E_j^S \tag{9}$$

$$E_{i,j}^{CH} = E_i^{CH} - E_j^{CH} \tag{10}$$

In Figs 3 and 4 the system components are represented by rectangles that are real units (or subsystems); the rhombuses and circles are fictitious units called junctions (J) and bifurcations (B), respectively, which are used to interconnect the subsystems.

The fuel and product definitions follow the SPECO approach [27] as follows: if the variation of specific exergy (or of its components with a positive contribution to the exergy definition) is positive throughout the process, this variation plus the exergy of flows of energy generated in the component define the product. On the other hand, if the variation of the specific exergy (or of its components with a positive contribution to the exergy) is negative throughout the process, this variation is added to the exergy of the energy flows supplied to the component in the input definition. The opposite happens with the components with a negative contribution in the exergy definition, such as the entropic component in the H&S Model. In this case, the H&S Model defines the productive flows of the entropic ($E_{5:1}^{E}$) and chemical ($E_{3:2}^{CH}$) components as input from the environment, and the entropic ($E_{5:1}^{E}$) component as a product, see Fig. 4.



Figure 3. Productive diagram - E Model.



Figure 4. Productive diagram - H&S Model.

3.1.2. Monetary cost balance

Figure 5 shows the monetary cost balance for the H&S Model, expanded in matrix form, which is obtained by applying the cost balance from Eq. (1) to each of the 5 subsystems (AC, CC, GT, RB, and ENV) and at enthalpic (J_{H} - B_{H}) and entropic (J_{S} - B_{S}) junctions-bifurcations of the productive diagram (Figure 4).

		Internal v	aluation	1		Cost		External	
		r)	4		- r	
0	0	$-E_{4:5}^{S}$	$-E_{5:1}^{S}$	0	$E_{2:1}^S + E_{3:2}^S + E_{4:3}^S$	L°JB-31		0 1	20
$-E_{3:2}^{H}$	0	0	0	$E_{3:4}^H + E_{4:5}^H + E_{5:1}^H$	0	C_{JB-H}		0	
$-E_{3:2}^{CH}$	0	0	$E_{5:1}^{S}$	$-E_{5:1}^{H}$	0	C _{ENV}		Z_{ENV}	
0	0	$E_{4:5}^{S} + Q_{U}$	0	$-E_{4:5}^{H}$	0	. C _{RB}	=	Z_{RB}	
0	$W_{AC} + W_N$	0	0	$-E_{3:4}^{H}$	$-E_{4:3}^{S}$	C _{GT}		Z_{GT}	
$E_{3:2}^H + E_{3:2}^{CH}$	0	0	0	0	$-E_{3:2}^{S}$	CCC		L_{AC}	
0	$-W_{AC}$	0	0	0	$-E_{2:1}^{S}$	E CAC 7		7 7	
1	$ \begin{array}{c} 0\\ E_{3:2}^{H} + E_{3:2}^{CH}\\ 0\\ 0\\ -E_{3:2}^{CH}\\ -E_{3:2}^{H}\\ \end{array} $	$\begin{array}{cccc} 0 & -W_{AC} \\ E_{3:2}^{H} + E_{3:2}^{CH} & 0 \\ 0 & W_{AC} + W_{N} \\ 0 & 0 \\ -E_{3:2}^{CH} & 0 \\ -E_{3:2}^{H} & 0 \end{array}$	$\begin{array}{cccccc} 0 & -W_{AC} & 0 \\ E_{3:2}^{H} + E_{3:2}^{CH} & 0 & 0 \\ 0 & W_{AC} + W_{N} & 0 \\ 0 & 0 & E_{4:5}^{S} + Q_{U} \\ -E_{3:2}^{CH} & 0 & 0 \\ -E_{3:2}^{H} & 0 & 0 \end{array}$	$\begin{array}{cccccccc} 0 & -W_{AC} & 0 & 0 \\ E_{3:2}^{H} + E_{3:2}^{CH} & 0 & 0 & 0 \\ 0 & W_{AC} + W_{N} & 0 & 0 \\ 0 & 0 & E_{4:5}^{S} + Q_{U} & 0 \\ -E_{3:2}^{CH} & 0 & 0 & E_{5:1}^{S} \\ -E_{3:2}^{H} & 0 & 0 & 0 \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$

Figure 5. Monetary cost balance in matrix form.

It is visible that the internal valuation matrix is composed of flows of the exergy components, power, and useful heat. It represents the process from the distribution of external resources to the formation of the final

products' cost. The cost matrix (or vector) is the modeling's unknown factor and is composed of the monetary unit cost of the flows generated in each of the subsystems. For instance, c_{AC} is the monetary unit cost of the compressor (AC) product, i.e., the flow $E_{2:1}^H$.

Due to the use of the equality criterion, some flows have the same unit cost. Examples are: $E_{3:2}^H$ and $E_{3:2}^{CH}$; $E_{4:5}^S$ and Q_U ; W_{AC} and W_N . In addition to those, all entropic component flows which leave J_H-B_H have the same unit cost as all entropic component flows leaving J_S-B_S.

The external valuation matrix contains the exergy of the fuel and its respective unit cost, plus the external hourly cost of each subsystem due to capital, and equipment O&M (Z). Because they are dummy components, the junction-bifurcations have zero Z-cost, as shown in the external valuation matrix in Fig 5.

The Z term, along with the device representing the environment in the diagrams, are key pieces in internalizing environmental costs in thermoeconomics. Figure 6 details this device and its input and output flows that are part of the monetary cost balance. The environmental device itself has no costs for acquisition, operation, and maintenance because it is a representation of the atmospheric environment itself. Nevertheless, in the case of the installation of some environmental treatment component (filter, electrostatic precipitator, among others) that generally is not represented in the physical diagram of the thermal system, the cost of this component can be internalized in the environmental device through the first two terms on the right-hand side of Eq. (4), and thus be redistributed to the entire system.

As for the cost of fines, environmental licenses, and permits, they should be internalized by Z_{env} , as well as the carbon pricing values. Nevertheless, the latter depends on whether it is revenue or expense in the carbon market.



Figure 6. Cost balance in the environmental device.

In the case of revenue, which can occur due to a reduction in emissions below the stipulated which generates a saleable credit, the term Z_{env} enters negative on the balance sheet. Since the environmental device closes the loop and redistributes the costs to the other equipment and final products in the plant, this credit reduces the other monetary costs and can influence the plant's production decisions.

On the other hand, an expense related to carbon credits, such as the need to buy credits since the company was not able to reduce emissions as stipulated, makes the term Z_{env} positive and similarly ends up increasing the costs of other internal flows and final products of the plant.

In summary, the equation shown in Fig. 6 is highlighted in the text as Eq. (11) and its analysis can be done as follows:

- The environmental device (ENV) has no hourly cost due to capital and O&M, but in the case of using environmental treatment equipment (which is generally not represented in the physical structure of the system), these terms can be considered within Z_{ENV} ;
- The environmental costs of licenses and permits are internalized through the environmental device term Z_{env} ;
- The costs associated with the carbon market are also internalized through the term Z_{env} . In the case of revenue, this term is negative and in the case of expenditure, it is positive.

In all three cases, as device ENV closes the loop (Figure 2), the costs are systematically redistributed to the other subsystems and consequently to the final products of the plant in the case of the H&S Model (Figure 4).

$$Z_{ENV} = Z_{cap} + Z_{O\&M} + Z_{env}$$

(11)

3.1.3. Results

Figure 7 represents a generic cogeneration (combined heat and power – CHP) system in which out of one fuel (Q_F), two products (W_N and Q_U) are generated, as is the case with the gas turbine system in Fig. 1. By applying the cost balance of Eq. (1) to this generic system, one obtains Eq. (12), in which c_{W_N} and c_{Q_U} are the monetary unit costs of the final products. Note that Eq. (12) is the equation of a straight line of the type y = A.x + B, and can be written according to Eq. (13).

$$c_{W_N} = -\frac{Q_U}{W_N} c_{Q_U} + \frac{c_F Q_F + Z}{W_N}$$

$$c_{W_N} = -A \cdot c_{Q_U} + B$$
(12)
(12)



(13)

Figure 7. Accounting flows in cogeneration.

Regardless of the applied thermoeconomic methodology, the solution to Eq. (12) will be an ordered pair of the monetary unit cost of the net power (c_{W_N}) and of the useful heat (c_{Q_U}). Some studies [13,28–30] have already compared several methodologies in problems of this type and confirmed that these ordered pairs belong to the same straight solution when the system has its operational conditions defined, such as the net power/useful heat ratio and the global exergetic efficiency.

Figure 8 represents possibilities for this straight solution generically. In all the possibilities, the higher the unit cost of power, the lower the unit cost of heat, and vice versa.

Considering the central straight line (solid blue line) as the specific condition for a cogeneration system, changes in the thermodynamic model move the straight line to new positions parallel to the initial one [28,31], as shown by the dashed lines in Fig. 8.



Monetary unit cost of heat $(c_{0_{II}})$

Figure 8. Unit cost solution line.

Figure 9 shows the monetary unit cost of the final products (ordered pair) of the cogeneration system for some situations. The values were obtained through the cost balance of Eq. 1 when applied to the diagrams of Figs 3 and 4 for E and H&S Models, respectively. For this second model, the balance is detailed (matrix form) in Fig. 5 to highlight the Z_{ENV} .

The points that belong to the central line, identified as the base case in the caption of Fig. 9, represent the case in which no carbon credit values are being internalized.

Note that the increase in emissions represents a reduction in the efficiency of the process and a consequent increase in production costs. Thus, the straight solution moves away from the origin and the costs of the final products increase. On the other hand, the reduction of emissions approximates the solution line to the origin, reducing the costs of the final products as a result of an improvement in the process's efficiency.

In order to analyze the expenses and revenues of the carbon market, some hypotheses were considered and realized. Starting from the base case, and considering that the increase in CO_2 emissions means that the system is emitting above the established, carbon credits must be purchased and thus an expense is generated for the plant. In the case of reduction/removal of emissions, the system generates a credit that could be sold and thus generate revenue. In the case of the H&S Model, the value of carbon credits is internalized in the environment device with ($Z_{env} > 0$) in the case of expenses and ($Z_{env} < 0$) for revenue. In E Model, internalization is done via CC with ($Z_{carbon credit} > 0$) in the case of expenses and ($Z_{carbon credit} < 0$) for revenue.

Hypotheses of increase (10% to 50%) and reduction (-10% to -50%) of emissions in relation to the base case were simulated. Table 4 shows the monetary unit costs of the final products ($k_{Q_U}^*$ and $k_{W_N}^*$) for all these

situations, in addition to the amount of carbon credits that these variations in emissions could generate and the costs (revenue and expense) associated.

When analyzing (c_{Q_U}) and (c_{W_N}) , it is verified that in E Model the costs of the final products vary approximately 5% and 25% for the cases of increase/reduction of 10% and 50% in the emissions compared to the base case. In the case of the H&S Model, for these same situations, the variations in the costs of the final products are approximately 5% and 26%. The variations in the costs obtained are due to the different criteria of each model, such as the internalization of carbon credits in the CC and the environmental device.

When analyzing the carbon credits for the simulated situations, it is observed that a 10% variation in emissions generates 5.3 credits/day which corresponds to an expense/revenue of \$455/day. In the most extreme case of variation (50%) in emissions, expenditure/revenue can reach \$2276/day. The use of value generated by the purchase or sale of the carbon credit can be used as an indicator for the decision-making of companies concerning the installation of environmental equipment or the purchase of carbon credits.

Remembering that this work aims to demonstrate the thermoeconomic methodology to be used for the inclusion of carbon pricing in cogeneration systems and not in the behavior of the system that generates an increase or reduction/removal in emissions, nor in the definition of the emissions parameters that will regulate the carbon market.



Figure 9. Monetary unit cost variation due to emissions.

	E Model		H&S Model				
Emissions -	E Woder		1100		 Carbon credit/day 	\$/dav	
	(c_{Q_U})	(c_{W_N})	(c_{Q_U})	(c_{W_N})			
+50%	120.33	87.66	102.57	96.32	26.7	-2273	
+40%	115.64	84.21	98.41	92.37	21.4	-1818	
+30%	110.95	80.76	94.26	88.42	16.0	-1364	
+20%	106.26	77.31	90.10	84.47	10.7	-909	
+10%	101.57	73.86	85.95	80.52	5.3	-455	
Base case	96.88	70.41	81.79	76.57	0	0	
-10%	92.19	66.96	77.64	72.62	5.3	455	
-20%	87.51	63.51	73.48	68.67	10.7	909	
-30%	82.82	60.06	69.33	64.72	16.0	1364	
-40%	78.14	56.6	65.17	60.77	21.4	1818	
-50%	73.45	53.15	61.02	56.82	26.7	2273	

Table 4. Monetary unit cost [\$/MWh] and carbon credit for the simulated situations.

Analyzing the inclusion of environmental costs, such as carbon pricing, one observes in Eq. (12) that the Z term, and consequently the Z_{env} , change the B coefficient of the straight line equation (Equation 13), and, therefore, would also shift the initial straight line (base case) to parallel lines compared to the initial condition. In the case of revenue ($Z_{env} < 0$), the straight line approaches the origin by decreasing product costs since revenue was generated from the sale of carbon credits. For expenses ($Z_{env} > 0$), the straight line moves away from the origin since it is necessary to buy carbon credits, that is, there is an increase in production costs. It is worth noting that the straight line moves to different but parallel positions. In any case, since coefficient A is not changed, the slope remains the same. Furthermore, by defining the system conditions and including the environmental cost, different thermoeconomic methodologies that consider the environmental device define ordered pairs of power and heat cost belonging to the same straight line solution.

4. Conclusions

This study described and detailed a thermoeconomic methodology to internalize monetary environmental costs in thermal system analyses through an example of a cogeneration system with a gas turbine.

The costs focused on the paper are the pricing of carbon emissions. However, the internalization of other environmental costs, such as the licenses, permits, and acquisition of environmental treatment/control equipment were also considered.

The cogeneration system was chosen because it is one of the main emitters of greenhouse gasses according to data from the International Energy Agency (IEA).

Models E and H&S were used. However, the focus is on H&S because it is a methodology that defines the device to represent the environment in the diagrams. This device is responsible for dissipating the cycle waste and has a key role in the internalization of environmental costs and in the systematized redistribution of costs for the remaining components and final products of the system. Nevertheless, any other thermoeconomic methodology, based on exergy, which coherently defines this environmental device, could be used following the same methods. Moreover, this model was proposed to take into account the treatment of waste (exhaust gases in the case of this work) and its costs, which are directly associated with pollutant emissions and, consequently, with the carbon market.

Since energy conversion systems generate environmental damage, their full analysis must take into account technical, economic, and environmental aspects to meet the environmental agenda signed in the Paris Agreement.

The current study presented the H&S Model as a feasible tool to reach this purpose by detailing how the carbon market can be taken into account and the pricing of carbon and other environmental costs internalized into the analysis. Besides detailing the calculation methodology, it also showed the behavior of the results of the cogeneration systems' final products' monetary costs. Incorporating costs associated with climate change into economic decision-making through carbon pricing can help encourage changes in production, consumption, and investment patterns. Thus, assisting in the energy transition process toward the planet's decarbonization.

This study concludes that the proposed methodology is coherent from the theoretical perspective of thermodynamics and thermoeconomics and can be used to allocate carbon credits to the internal and final products of thermal systems.

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Nomenclature

- AC Air compressor
- c monetary unit cost [\$/MWh]
- CC Combustion chamber
- CEPCI Chemical Engineering Cost Index
- CHP Combined heat and power
- E Exergy Flow [kW]
- ENV environmental device
- GHG Greenhouse gas
- GT Gas turbine

- IPCC Intergovernmental Panel on Climate Change
- JB Junction-bifurcation
- *k** Exergetic unit cost [kW/kW]
- Q Heat (exergy) [kW]
- RB Recovery boiler
- W Power [kW]
- Y Generic thermodynamic magnitude [kW]
- Z Hourly equipment cost [\$/h]

Greek symbols

 λ specific CO₂ emission [g/MWh]

Subscripts and superscripts

- 0 Reference conditions
- CH Chemical exergy [kW]
- Env environmental
- F Fuel
- H Enthalpic flow [kW]
- i;j Indexes for productive components
- in Inlet
- N Net
- out Outlet
- PH Physical exergy [kW]
- S Entropic flow [kW]
- U Useful heat

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