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# Innovative indicators for quantifying energy flexibility of district heating networks

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## Abstract:

This article proposes a methodology to quantify the flexibility potential of a district heating network. By "flexibility potential" is meant here the degree of freedom in combining and controlling the energy units of a district heating network in order to meet a target demand. This potential has been modelled as the logarithm of the available combinations (or configurations), like the Shannon/Boltzmann entropy in statistical mechanics. The methodology produces a set of four entropy-inspired indicators. The first two, called "structural" and "operational" flexibility, are proportional to the number of available combinations of energy units and their input and output thermal powers, respectively. The other two indicators are the "effective" counterparts of structural and operational flexibility, and indicate the extent to which a given demand profile exploits the overall flexibility potential of a district heating network. The methodology takes into account the different sources of flexibility related to heat production, (de)storage, dissipation or diversion, as well as load adjustment techniques (demand management). Given its scaleless and dimensionless formulation, the methodology can be applied to energy systems of any size or energy type. The methodology is presented with the open-source tool used for the calculations and an illustrative example.

## Keywords:

District heating, District heating networks, Energy flexibility, Operational flexibility, Thermodynamics, Entropy.

## 1. Introduction

The phenomenon of flexibility has been a source of many definitions and ambiguity in many fields of research. With respect to energy networks in the broadest sense, the International Energy Agency gave a first formal definition of "flexibility" in 2011: the ability of an energy system to react to temporal changes in energy production and demand [1].

The first active research on the subject of energy flexibility focused on power systems. The trigger for such issues was the decarbonization of these networks [2], which often leads to the inclusion of renewable sources in the energy mix [3], e.g., solar or wind power. Thus, possible sources of flexibility in electricity networks have been identified for a long time [4].

Although the question of flexibility in heat networks came later, several sources of flexibility have already been identified in the past, often with the intention of bringing flexibility to electricity networks [5]. The classical approach is to implement thermal energy storage, which can consist of centralized [6] or decentralized [7] collective physical storage units, or the use of the thermal inertia of the network itself [8], although the latter approach has its limitations in new generation (low temperature) networks [9]. Other approaches consist in controlling water flows [10] or their temperatures [7].

Thermal networks are generally considered as a way to increase the flexibility of electrical networks. For example, power-to-heat (P2H) systems increase flexibility because they allow excess electricity generated by PV panels to be converted into heat [11]. It is perhaps because of this "grid support" role that research on their intrinsic flexibility is less advanced. In addition, it is often evaluated indirectly, through a host of indicators that are related to flexibility: the ratio of renewables in the energy mix [12]; operational maps expressing the relationship between active and reactive power in an electrical network [13]; operational maps linking combined heat and power production in CHP units to the interface between electrical and thermal networks (see Fig. 5 in [9]); or the economic cost of improving flexibility, e.g., through better control devices [14].

While all the above approaches are useful and interesting, the authors of this paper believe that the field of heat networks lacks explicit and quantitative flexibility indicators. Similarly, the diversity of available energy units is an important source of flexibility, both for the design and the management of networks, yet it is not well known [15] and apparently lacks quantitative indicators. Finally, the authors also believe that efforts are needed to propose indicators that are decoupled from annual simulations and other indicators such as economic ones,

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as these two factors prejudge the control strategy. Only when dedicated flexibility indicators with their own formulation and magnitude (or even units) are available, can network flexibility be accurately analyzed.

This article proposes an interpretation of flexibility inspired by the notion of entropy in statistical mechanics. That is, flexibility is understood as the logarithm of the possible combinations between the different energy units of a network. This notion is broken down into combinations of units (so-called "structural" flexibility) and combinations in the management of units (so-called "operational" flexibility). An explicit and non-dimensional indicator, proportional to the number of combinations, is proposed for each declination. The analysis allows to take into account controllable or non-controllable production units (renewables), thermal storage (physical unit or grid inertia), consumption or thermal dissipation, or demand-side management (DSM).

The paper is structured as follows: Section 2 describes in detail the method and the calculation of the indicators; Section 3 presents an illustrative case with its results and analysis; Section 4 discusses several aspects of the method that could be improved; and Section 5 summarizes the most relevant insights about the method and the illustrative case.

## 2. Materials and method

### 2.1. Hypotheses and data pre-treatment

This methodology is based on the first order thermodynamic analysis, and thus on the following assumptions and postulates:

- The system is in pseudo-permanent regime. Dynamic considerations are not taken into account. When the user declares a thermal unit, they must make sure that it can deliver heat at the exact moment required by the end-users. Any temporal mismatch due to ramp-up or ramp-down times, or inertia in general, must be ruled out before supplying a unit’s thermal power range to the method. Taking into account temporal variability is a perspective for the authors, that will be treated in later researches.
- All units considered in the analysis are capable of producing heat at the temperature required by the demand profile. Issues related to temperature mismatch are not modelled. When the user declares a thermal unit, they must make sure that it can deliver heat at the temperature required by the end-users. Any decrease in temperature due to transport losses or heat exchange must be discounted from the unit’s thermal output before supplying this information to the method.
- Spatial considerations are not taken into account. When the user declares a thermal unit with a power range, they must make sure it is the power range that the unit can deliver on the end-user side. Therefore, any heat losses due to transport must be discounted from a unit’s power output before supplying this information to the method.
- Head loss along pipes is neglected.

The different sources of flexibility in a network are modelled as thermal power ranges. Table 1 shows how equivalence can be established for different flexibility sources.

**Table 1.** Equivalent thermal units considered by the method, with their interpretations and some examples.

| Equivalent Thermal Unit | Interpretation | Example of a real unit |
|---|---|---|
| Adjustable production | A production unit whose thermal power output can be adjusted by its operator. | Biomass boiler |
| Non-adjustable production | A production unit whose thermal power output cannot be controlled by its operator. | Solar thermal collector field |
| Storage/de-storage | Any means of accumulating heat. It can be a physical unit or a management technique. | Thermal storage tank |
| Dissipation/Diversion | To displace heat outside of the system’s boundaries. | Heat dissipation by aerothermal units. Heat exports to another aggregator. |
| Demand-Side Management | To act upon the demand profile in order to modify it for a better matching with the available thermal productions. | Using reward mechanisms in order to direct users’ energy consumption to target periods of the day. |

### 2.2. Underlying concept of the method

The indicators proposed in this article are inspired by the notion of entropy. In thermodynamics, if all possible configurations of a thermodynamic system have the same probability of existing, entropy is determined through Boltzmann’s formula: $S = \ln(\Omega)$. The authors of this article took inspiration on that approach, and suggest the concept of ‘combinatorial multiplicity’ in order to analyze the flexibility of district heating networks.

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Combinatorial multiplicity (*Ω*), an analogy to thermodynamic multiplicity, is understood as the number of possible ways to operate the units of energy in a district heating network given an external constraint. In this article, the external constraint is a thermal power demand.

The authors implemented this concept of flexibility through two approaches. One, called "structural flexibility", relates to the number of possible combinations of energy units that satisfy the demand. The other, called "operational flexibility", is related to the number of combinations of thermal energy outputs within the combined energy units. Figure 1 summarizes the two approaches visually, and the following subsection explains the calculation procedure in a formal and detailed manner.

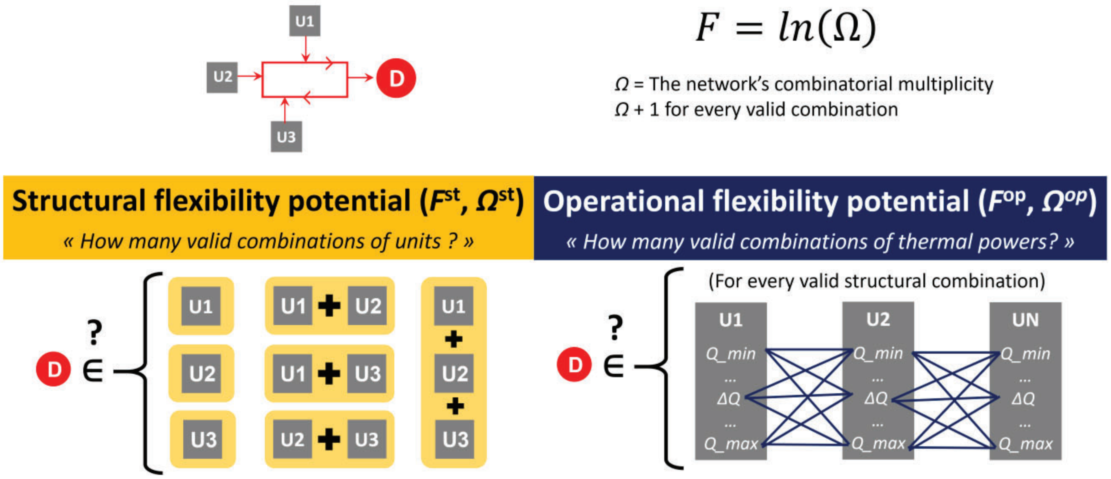


**Figure 1.** Flexibility potential assessment with a combinatorial approach inspired by the concept of entropy: structural flexibility (down, left) and operational flexibility (down, right).

## 2.2. Structural flexibility

Let us consider a heating network equipped with *N* equivalent units from the list provided in Table 1. The minimum and maximum heat outputs of the network are the sum of the minimum and maximum specific heat outputs of all units (1).

$$\dot{Q}_N^{\dagger} = \sum_{n=1}^{N} \dot{Q}_n^{\dagger} \qquad \langle \dagger = min, max \rangle \tag{1}$$

For the flexibility analysis, the global control step of the network has been defined as the greatest common divisor of the control steps of all units (2). This global control step, together with the global minimum and maximum power, determines the operating range of the network (3).

$$\Delta\dot{Q}_N = gcd\left(\Delta\dot{Q}_{j=1}, \dots, \Delta\dot{Q}_{j=J}\right) \tag{2}$$

$$Q_N \in \left[\dot{Q}_N^{min} : \Delta\dot{Q}_N : \dot{Q}_N^{max}\right] \tag{3}$$

The structural flexibility is evaluated over this operating range, power by power. All possible combinations of units are considered, from 1 to N selected units. For each combination of units, if the requested power is well within the combined operating range, the number of local configurations at that power ($\Omega_q^{st}$) increases (4).

$$\forall\, Q \in Q_N \; and \; \forall\, st \in C^{st} \qquad if\ Q\ \in Q^{st} \;\Rightarrow \Omega_q^{st} = \Omega_q^{st} + 1 \tag{4}$$

The overall structural configuration number (Ωst) is the sum of all local structural configuration numbers over the operating range of the network (5). Because all combinations are assumed to be equiprobable, the structural flexibility is determined by a parallelism of the Boltzmann equation (6).

$$\Omega^{st} = \sum_q \Omega_q^{st} \tag{5}$$

$$F^{st} = ln(\Omega^{st}) \tag{6}$$

A value of $F^{st} = 0$ indicates a choke point, i.e., a thermal power demand that no structural combination of energy units can satisfy. An $F^{st}$ of one indicates a structurally rigid point, i.e., only one structural combination can satisfy the demand. Note that a combination does not necessarily mean a single unit; it can be a combination of several units. An $F^{st}$ greater than one means that the network is structurally flexible, i.e., several combinations of units can meet the demand. The higher the $F^{st}$ value, the more flexible the network is. These considerations apply to both local and global flexibilities.

## 2.3. Operational flexibility

Operational flexibility is concerned with the combinations of driving units, rather than the combinations of units. Indeed, often the same combination of units can be driven in different ways to satisfy the same demand. The number of operational configurations is determined in a similar way to the structural one, with one difference: each valid combination of thermal outputs counts (7).

$$\forall\, q \in Q_N \wedge \forall\, c^{st} \in C^{st} \qquad \forall\, q^{st} = q \implies \Omega_q^{op} = \Omega_q^{op} + 1 \tag{7}$$

A valid combination is one that satisfies the power demand even with overproduction, provided that the overproduction can be managed (i.e. stored or dissipated). It is important that there is no unmanaged overproduction. Operational flexibility is the logarithm of the number of operational configurations (8 and 9).

$$\Omega^{op} = \sum_q \Omega_q^{op} \tag{8}$$

$$F^{op} = ln(\Omega^{op}) \tag{9}$$

The structural and operational analyses, applied over the entire operating range of a network as defined by (4), lead to two distributions of flexibilities (structural and operational). Structural flexibility and operational flexibility provide non-redundant information with respect to each other. Structural flexibility does not take into account whether the units operate at their limits (upper or lower). Hence the need to define operational flexibility. Both give rise to different distributions of multiplicity, and their maximum will often be reached at different thermal powers. When combined, they make it easier to forecast choke points in the operation of a district heating system.

For clarity, a calculation example of both flexibilities is offered in Fig. 2. Let a district heating network feature three different thermal units. Each one has its own adjustable operating range. Then, the network is faced to a thermal power demand that falls within its production range. It is probable that the demand can be covered by combining the production units in different ways. Here, the term ‘combination’ is understood as ‘operating one or multiple thermal units simultaneously’. Following this logic, structural flexibility considers how many combinations of units can cover the demand *at least once*. Operational flexibility considers how many different ways of operating the units exist in order to cover a demand. In operational flexibility, every valid combination counts. In structural flexibility, every valid combination of units counts only once, independently of its number of valid operational combinations (see Fig. 2).

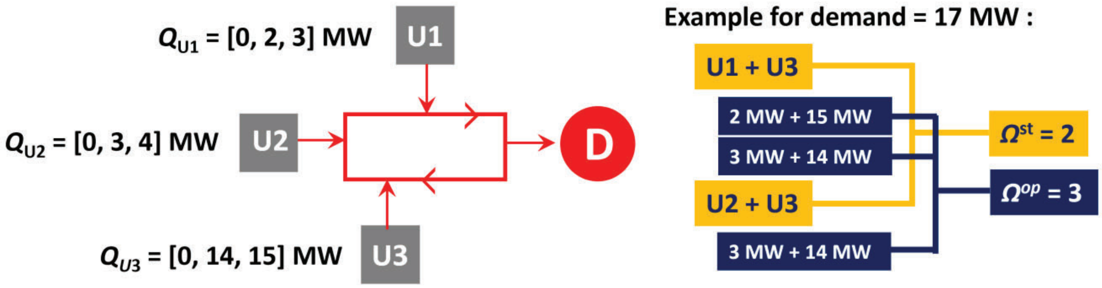


**Figure 2.** Calculation example of structural and operational multiplicities.

Let readers note that in order to count as a valid solution, a combination must satisfy the demand exactly. This logic applies in both structural and operational flexibility analyses. Therefore, in structural analysis, it is not sufficient that the demand be within a combination’s aggregated operating range. The structural combination must have at least one operating point where the thermal power output equals the demand exactly. This means that the discretization of thermal power ranges is an important point of the methodology. This aspect is discussed in section 4.

## 2.3. Dedicated open-source Python tool for the calculations

An open-source tool has been written in Python 3.9 for the implementation of the methodology and the calculation of the illustrative case presented in section 3. The tool allows to introduce a series of lists to declare the equivalent energy units in the system (adjustable production, non-adjustable production, storage, dissipation or DSM). Lists are used to declare unit names, operating ranges, and operating steps. Prohibited combinations of units may also be declared, in case the practical constraints of the case study do not allow the simultaneous operation of certain units. Then the tool starts with a routine to check the correctness of the input data (for example, that the operating steps are consistent with the operating ranges). After this check, a dictionary of unit combinations is created and purged of all user-defined forbidden combinations. Then, the tool uses several "for" loops and "if" statements to analyze each combination and check if it complies with the energy balance and if it corresponds to a target demand value. If so, this combination is considered flexibility. The target value can be 1) either a user-defined request; 2) or each stage of operation of the neighborhood network as a whole, if the user requires a complete distribution of flexibility. After this process, the tool creates a dictionary of valid combinations and calculates flexibility indicators. After the whole process, the tool returns the numerical values of the flexibility for the network and plots the distributions of the structural and operational flexibilities (i.e., the figures presented in section 3). The tool is encoded in UTF-8 and requires the "itertools", "matplotlib" and "numpy" packages.

## 3. Results for an illustrative case

The previous section described innovative indicators of structural and operational flexibilities and how to evaluate them. This section shows how to apply the methodology and perform a detailed flexibility analysis of a heat network. For this purpose, a simple illustrative example has been developed to show all important information in the flexibility analyses. Readers are reminded that the actual output of this article is the methodology itself, not this illustrative example.

### 3.1. System description and input data

The illustrative example consists of a heat network with 6 units (Table 2). As a reminder, these are equivalent units, which can represent physical devices, a complete process/machine or an energy management technique.

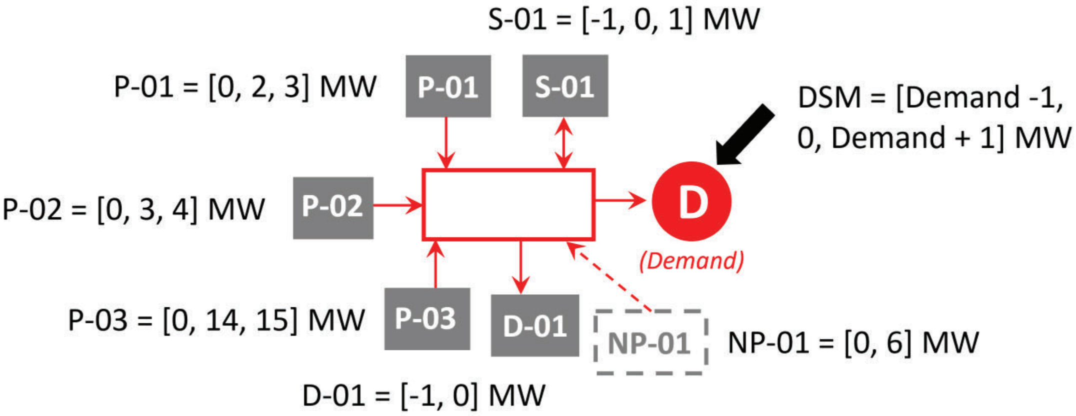


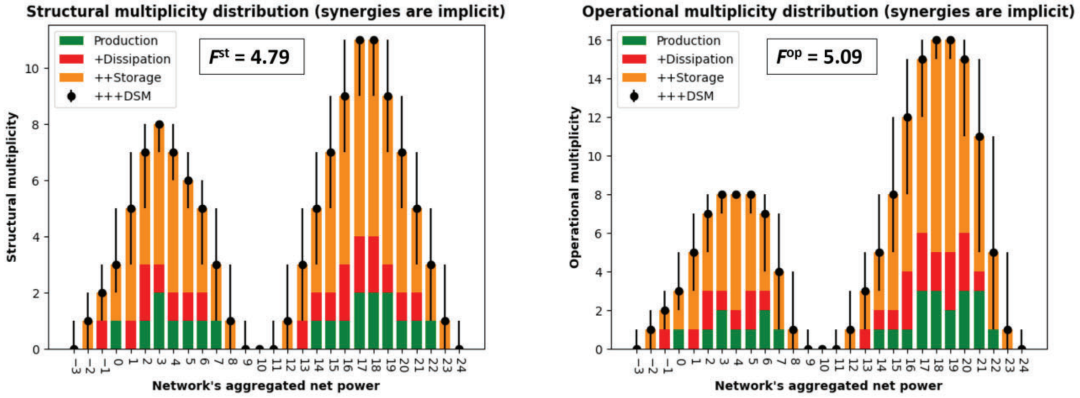
**Figure 3.** Schematical representation of the illustrative case.

**Table 2.** Equivalent thermal units considered in the illustrative case.

| Unit name | Real unit | Equivalent thermal unit | Operating range |
|---|---|---|---|
| P-01 | Biomass boiler | Adjustable production | [0, 2, 3] MW |
| P-02 | Heat pump | Adjustable production | [0, 3, 4] MW |
| P-03 | Heat import | Adjustable production | [0, 14, 15] MW |
| NP-01 | Solar collector field | Non-adjustable production | [0, 6] MW |
| D-01 | Aerothermal dissipation | Dissipation/Diversion | [-1, 0] MW |
| S-01 | Thermocline storage unit | Storage/De-storage | [-1, 0, 1] MW |
| DSM | Tariff-incentivized modification of users' consumption patterns | Demand Side Management | [Demand -1, 0, Demand +1] MW |

### 3.2. Analysis of structural and operational flexibility

Figures 4 and 5 show the so-called structural and operational flexibilities distributions, respectively. Both are to be represented on the aggregated power range of the network (x-axis). In this example, the lower bound (-3 MW) corresponds to 0 MW generated (P-01, P-02, P-03), 1 MW stored (S-01), 1 MW dissipated (D-01) and 1 MW shed (DSM). The upper bound (+24 MW) corresponds to 22 MW generated (P-01 = +3 MW, P-02 = +4 MW, P-03 = +15 MW), 1 MW de-stored (S-01) and 1 MW anticipated (DSM).



(a) (b)

**Figure 4.** Flexibility distributions over full demand range: a) Structural flexibility, b) Operational flexibility.

The y-axis represents local flexibility at each power, determined by the methods described in Sections 2.2 (structural) and 2.3 (operational). The contribution of the generation units (green bars) is represented at the base of the distribution because it does not depend on the other unit types. Then, the dissipation units are represented in combination with the production (red bars), because they can only remove from the system heat already contributed by other units. The contribution of the storage units is shown in combination with the productions and dissipations (orange bars). It depends on a previous production (which could be partially dissipated), but also on the state of charge of the storage units. The asymmetric error bars represent the effects of the DSM techniques, once the other units are considered.

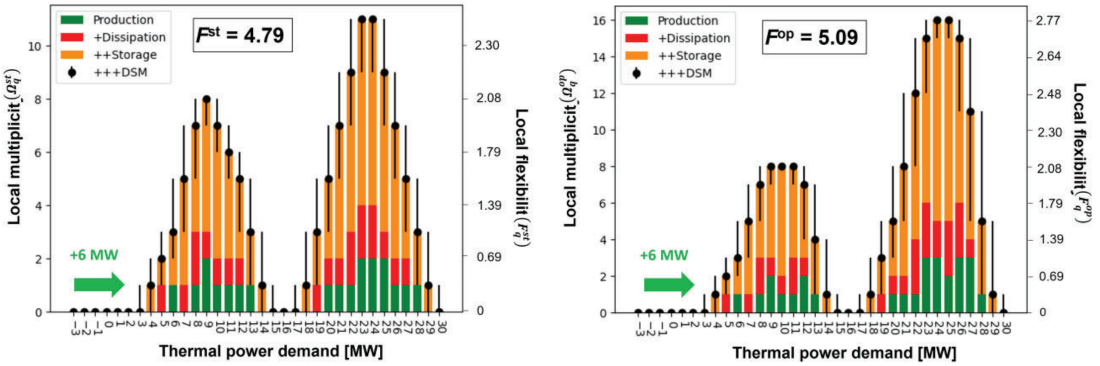
Thus, it turns out that the controllable generation units will determine the overall flexibility range to a large extent; the other units only expand the flexibility range implemented by the productions. Dissipation widens it only to the left, while (de-)storage widens it to the left and to the right. The DSM, rather than providing flexibility, shifts demand to other powers, which will be advantageous or not depending on the local flexibility of neighbouring powers.

In this illustrative case, the outputs allow two ranges of flexibility: [2 - 7] MW and [14 - 22] MW, which result from all possible combinations between P-01, P-02 and P-03. Then, adding dissipation widens them to 1 MW and 13 MW respectively, and allows to reach -1 MW if only dissipation is used. Storage extends the ranges further, to -2 MW or 12 MW under load, or to 8 MW or 23 MW under discharge. In addition, the storage load would have allowed -1 MW, 1 MW, and 13 MW even with no dissipation. And of course, dissipation and storage increase the local flexibility on the powers already covered by the productions. In fact, they include cumulative synergies. Quantifying and locating precisely the different synergies is an interesting perspective foreseen by the authors.

The potential effects of DSM (error bars on the y-axis) represent the largest possible change in F in the DSM range from a specific power. DSM can shift demand to powers where the local flexibility of the grid would be different. For example, the local structural flexibility at 4MW may switch between 1.79 and 2.08, as the DSM of +/- 1MW would shift demand to 5MW or 3MW respectively. Both the increase and decrease in flexibility are displayed, as the DSM is not always controllable. For example, time-shifted heating requirements often have to be made up later. The DSM can give flexibility to certain power demands that are not accessible by other units (-3 MW, 9 MW, 11 MW and 24 MW in this example). If a power unit is in a flexibility plateau (e.g., 10 MW), DSM will have no effect on its flexibility. DSM will not cause flexibility to rise above its overall maximum, nor will it cause it to fall below zero.

Any power that may fall to zero flexibility due to a poorly managed DSM is a potential choke point. Any power slice that has zero flexibility, but can be increased by DSM, is a bypassable choke point. Any slice of power with zero flexibility without a solution is a hard choke point.

The effects of non-drivable power generation (e.g., renewables) are displayed in Figures 3 and 4. A non-drivable generation of 6 MWth has been assumed. This could symbolize, for example, the installation of a solar thermal collector field with a peak generation of 6 MWth. The new flexibility distributions are given in Fig. 5.



(a) (b)

**Figure 5.** Flexibility distributions after consideration of renewables (non-adjustable production): a) Structural flexibility, b) Operational flexibility.

As a reminder: non-modulable generation is modeled as a heat input to the network that cannot be modulated in power. It must be either used, stored or dissipated. As a result, the two flexibility distributions are shifted to the right on the x-axis (see Figures 4a and 4b). While the old distributions went from -3 MW to 24 MW (Fig. 1), the new ones go from 3 MW to 30 MW. The old lower bound of -3 MW has been retained in the figure for illustrative purposes, but note that there is no flexibility before 3 MW.

Since non-adjustable heat generation is a "must" unit, there is no freedom to (not) select it. Therefore, this does not change the number of possible combinations. Note that there are still 32 combinations of units and 160 combinations of input/output heat outputs (see Appendix A), as in the previous scenario. The only change is that now all possible combinations include non-adjustable generation, and thus have +6 MW of heat output.

The effect of non-controllable productions may seem harmless, but it is not. When the district network is confronted to a demand profile, it will be forced to operate at certain regions of its flexibility distribution. The integration of new, non-adjustable thermal productions (like renewables) may completely unbalance the flexibility matching of the network. In other words, the effects of renewables on flexibility are difficult to forecast even if the network had been well-designed for a demand profile. The methodology presented in this paper (and its calculation tool) may be used for early detection of flexibility issues due to changes in the energy mix of a territory.

# 4. Discussion on the method

## 4.1. On the combinatorial approach

The main advantage of a combinatorial approach is that all possible configurations of the system are taken into account. This is the reason why the authors speak of "flexibility potential", instead of "flexibility". A dynamic simulation can only give a "circumstantial" idea of the flexibility of a network, because it prejudges the control strategy, and therefore may encounter difficulties not necessarily due to the configuration of the network at the base. On the other hand, the computation time is the weak point of a combinatorial approach, especially if the number of units is higher than 15.

## 4.2. On disregarding other performance criteria

This methodology and its indicators are free of preconceived ideas about optimal management or control strategies. Criteria such as energy efficiency, exergy efficiency, economic performance, environmental effects... are totally ignored. The methodology does not give less importance to a specific choice of technical units according to their abundant need of primary energy, or high implementation costs, or high CO2 emissions, or long start-up/shutdown times... In fact, none of these parameters are needed as input to the model. Any combination that satisfies the demand while respecting the energy balance is an equally valid solution.

Heat quality (temperature) considerations are also very limited. Any solution that meets the final heat demand at the requested temperature is valid, regardless of the upstream temperatures. For example, two generating units producing heat at 120°C and 50°C respectively, and both meeting heat requirements at 35°C, are also valid and important under this methodology. Although higher production temperature may indicate higher transmission losses or lower energy efficiencies in substations. In any case, the user must ensure that any generation, storage, dissipation or load control unit included in the analysis can satisfy the heat demands at the requested temperature.

The methodology does not rule out solutions that may seem counter-intuitive or unreasonable from the perspective of first-order thermodynamic analysis. This concerns in particular the overproduction of heat followed by the dissipation of the excess. For example, assume a generating unit and a dissipating unit with thermal power ranges Q_prod = [0, 1, 2, 3] MW and Q_diss = [-1, 0], respectively. If the demand is Q_dem = 2 MW, it is equally valid to produce Q_prod = 2 MW or to produce Q_prod = 3 MW and then to dissipate Q_diss = -1 MW. The methodology gives equal weight to both possibilities. Any arrangement of units that exactly matches the demand is considered flexibility.

All of these are deliberate choices by the authors, in order to create indicators that focus solely on combinatorial flexibility. There are also many methods and tools for multi-criteria analysis of heating networks, with a wide variety of criteria. As for counter-intuitive solutions, it is precisely the variety of criteria that justifies not discarding them. The optimality of a solution largely depends on the context and the decision criteria. Overproduction followed by partial dissipation may seem unreasonable, but it can allow the purchase of primary energy at better rates, for example. In this case, it can become economically optimal. If an exploitation decision is possible, it can become optimal under certain specific conditions and must therefore be taken into account.

### 4.3. On discretizing continuous ranges of thermal production

The discretization of continuous ranges of thermal power is probably the most debatable aspect of this methodology. Any thermodynamic system is indeed continuous, and trying to analyze it on finite increments may seem too reductionist an approach. The best answer of the authors to this remark at the moment is: control. District heating networks are equipped with demand-oriented energy systems and often subject to some type of control, often discrete. The thermal powers in this method should be considered as control steps.

### 4.4. On applying the method to other types of energy networks

The methodology, as presented in this article, is intended and more suitable for district heating networks with sources of flexibility from this list: thermal energy production (including partial load operation); storage and retrieval of thermal energy; dissipation (or diversion) of thermal energy; and any demand management techniques that impact thermal energy demand profiles. Only sources of flexibility that can be "translated" into thermal power can be modelled.

Nevertheless, with some data pre-treatment by the user, its applicability can be extended slightly. First, the methodology (and the corresponding Python tool) allows to the user to set the parameters. Thermal power was used in this article, but electrical power could be used to replicate the analysis for electrical networks. Since the methodology is based on balance sheets, the analysis remains consistent as long as the input data is consistent. The same could be done with district cooling networks, as long as their sources of flexibility can be routed as cooling powers. It could even be applied to gas networks, choosing mass flow rates as units.

Also, the methodology is not necessarily limited to the district level. As the notion of spatiality is not modelled, the methodology can be applied to micro-grids (mW) as well as very large-scale (GW) grids, provided that the sources of flexibility are modeled accordingly.

Furthermore, the sources of flexibility taken into account by the methodology are described in such a generic way that they can represent many types of real units/techniques. For example, a conversion unit can be modeled as equivalent production and dissipation units.

The authors are aware that the aforementioned adaptations, although feasible, represent a burden for the user. Their future work is to integrate as many of them as possible into the methodology (and by extension, into the open-source tool), as new features.

Using natural logarithms for quantifying these indicators becomes very useful when expanding/connecting networks. For example, let us assume that a district heating network is connected upstream with the electric grid via electrically-driven mechanical-compression heat pumps. When the heat pumps are implemented, the combinatorial multiplicity of the whole system is the product of the combinatorial multiplicities of the grid and the district heating network. At the same time, the authors would like the indicator of flexibility to be the sum of the flexibilities from the grid and the heating network. Using logarithms allows both addition properties.

## 5. Conclusions and perspectives

The district heating domain lacks explicit and quantitative indicators to describe the flexibility potential apart from other performance criteria. In this paper, the authors provide an answer to this deficiency through a combinatorial approach. It understands the flexibility potential as the degree of freedom in the selection and control (separately or simultaneously) of the energy units of a network in the face of various demands. The following findings and perspectives can be highlighted:

- The method can help in the energy planning of urban energy networks. Namely, it can help anticipate potential choke points, or the possible effects of renewable energy deployment on existing networks.
- It is planned to study the possibility of working with continuous distributions of flexibilities, rather than discrete ones.

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- It is planned to study the applicability of the method to other energy networks, such as electricity networks, district cooling or gas networks. The dimensionless formulation of the method allows (in theory) to extend its application.
- It is planned to define additional indicators, for example an "effective" flexibility that would evaluate to what extent a given demand profile exploits the flexibility potential of the requested network.

The objective is to make the methodology as applicable as possible without putting the burden of data pre-treatment on the user.

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## Appendix A

Table A.1 shows the detailed analysis of all possible combinations of units and their thermal powers for the illustrative case shown in Section 3 of the main part of the manuscript. DSM was not processed in the combinatorics because it is represented on the final distributions of flexibility.

**Table A.1.** Complete list of structural combinations, and their operating ranges, in the illustrative case.

| | Combined units | Thermal power range of each unit involved [MW] |
|---|---|---|
| #1 | No units | [0] |
| #2 | ['P-01'] | [2.0, 3.0] |
| #3 | ['P-02'] | [3.0, 4.0] |
| #4 | ['P-03'] | [14.0, 15.0] |
| #5 | ['D-01'] | [-1.0] |
| #6 | ['S-01'] | [-1, 1] |
| #7 | ['P-01', 'P-02'] | [[2.0, 3.0], [3.0, 4.0]] |
| #8 | ['P-01', 'P-03'] | [[2.0, 3.0], [14.0, 15.0]] |
| #9 | ['P-01', 'D-01'] | [[2.0, 3.0], [-1.0]] |
| #10 | ['P-01', 'S-01'] | [[2.0, 3.0], [-1, 1]] |
| #11 | ['P-02', 'P-03'] | [[3.0, 4.0], [14.0, 15.0]] |
| #12 | ['P-02', 'D-01'] | [[3.0, 4.0], [-1.0]] |
| #13 | ['P-02', 'S-01'] | [[3.0, 4.0], [-1, 1]] |
| #14 | ['P-03', 'D-01'] | [[14.0, 15.0], [-1.0]] |
| #15 | ['P-03', 'S-01'] | [[14.0, 15.0], [-1, 1]] |
| #16 | ['D-01', 'S-01'] | [[-1.0], [-1, 1]] |
| #17 | ['P-01', 'P-02', 'P-03'] | [[2.0, 3.0], [3.0, 4.0], [14.0, 15.0]] |
| #18 | ['P-01', 'P-02', 'D-01'] | [[2.0, 3.0], [3.0, 4.0], [-1.0]] |
| #19 | ['P-01', 'P-02', 'S-01'] | [[2.0, 3.0], [3.0, 4.0], [-1, 1]] |
| #20 | ['P-01', 'P-03', 'D-01'] | [[2.0, 3.0], [14.0, 15.0], [-1.0]] |
| #21 | ['P-01', 'P-03', 'S-01'] | [[2.0, 3.0], [14.0, 15.0], [-1, 1]] |
| #22 | ['P-01', 'D-01', 'S-01'] | [[2.0, 3.0], [-1.0], [-1, 1]] |
| #23 | ['P-02', 'P-03', 'D-01'] | [[3.0, 4.0], [14.0, 15.0], [-1.0]] |
| #24 | ['P-02', 'P-03', 'S-01'] | [[3.0, 4.0], [14.0, 15.0], [-1, 1]] |
| #25 | ['P-02', 'D-01', 'S-01'] | [[3.0, 4.0], [-1.0], [-1, 1]] |
| #26 | ['P-03', 'D-01', 'S-01'] | [[14.0, 15.0], [-1.0], [-1, 1]] |
| #27 | ['P-01', 'P-02', 'P-03', 'D-01'] | [[2.0, 3.0], [3.0, 4.0], [14.0, 15.0], [-1.0]] |
| #28 | ['P-01', 'P-02', 'P-03', 'S-01'] | [[2.0, 3.0], [3.0, 4.0], [14.0, 15.0], [-1, 1]] |
| #29 | ['P-01', 'P-02', 'D-01', 'S-01'] | [[2.0, 3.0], [3.0, 4.0], [-1.0], [-1, 1]] |
| #30 | ['P-01', 'P-03', 'D-01', 'S-01'] | [[2.0, 3.0], [14.0, 15.0], [-1.0], [-1, 1]] |
| #31 | ['P-02', 'P-03', 'D-01', 'S-01'] | [[3.0, 4.0], [14.0, 15.0], [-1.0], [-1, 1]] |
| #32 | ['P-01', 'P-02', 'P-03', 'D-01', 'S-01']} | [[2.0, 3.0], [3.0, 4.0], [14.0, 15.0], [-1.0], [-1, 1]] |

## Nomenclature

| | |
|---|---|
| $\mathrm{C}$ | Combinations |
| $\mathrm{F}$ | Flexibility |
| $gcd$ | Greatest common divisor |
| $\mathrm{N}$ | Total number of thermal management units in the district heating network being analysed |

| | |
|---|---|
| n | n-th energy unit in the district heating network being analysed |
| $Q$ | Heat, or thermal power range (MW) |
| st | Structural combination of units (i.e., simultaneous operation of several units) |
| $T$ | Temperature |

**Greek symbols**

| | |
|---|---|
| $\Delta$ | step or increment |
| $\sum$ | summation |
| $\Omega$ | combinatorial multiplicity of the district heating network being analysed |

**Subscripts and superscripts**

| | |
|---|---|
| $D$ | thermal power demand range |
| $d$ | thermal power demand |
| $j$ | a particular combination (structural or operational) of thermal units in a district heating network |
| $min$ | minimal |
| $max$ | maximal |
| $op$ | operational |
| $st$ | structural |

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