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Metaheuristic vs. mathematical optimization: A comparison of methods for the design optimization of residential building energy systems

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Abstract:
The energy system of residential buildings and their impact on the transition towards an emission-free energy supply has been a focus in a wide range of studies over recent years. For the design of energy systems, a variety of methods are used, most commonly heuristic, mathematical optimization and metaheuristic approaches. While the strengths and weaknesses of these methods are well known, knowledge about the discrepancy in results produced for the design of energy systems is limited. Moreover, metaheuristics have rarely been utilized in the field of household energy system planning. This leads to problems whenever findings from different studies are compared and raises the question about the optimal choice of methodology under given circumstances. To approach this question, we examine the energy system of a residential building with two different methods - a mathematical optimization and a metaheuristic optimization applied to the same MILP model. The energy system model considers a PV system, a heat pump, a heat and a battery storage system as well as a gas boiler. The layout and size of these components along with their operation are optimized. We compare the results regarding the difference in layout and size of individual components, investment costs, operational costs, CO_2 emissions and computational performance of the methods. In this case study, the mathematical optimization resulted in the best Pareto front. Using the metaheuristic approaches, it is possible to compute a Pareto front in a considerably shorter time. However, the quality of the Pareto front is significantly worse.

Keywords:
Design Optimization, Energy System Planning, Metaheuristic, Mathematical Optimization, MILP, Residential Building.

1. Introduction

The design and operational optimization of energy systems, considering renewable energies and the resulting temporal resolution, is a complex mathematical problem. The complexity of such optimization problems increases strongly with an increasing number of variables, local optima, and non-linearities, which also leads to increasing computational effort [1]. In general, global optimization algorithms can be divided into two methods: Deterministic methods also called mathematical optimization and probability-based methods referred to as metaheuristic optimization. In the past energy systems and especially unit commitment problems were mostly modeled as Linear Programming (LP) or Mixed Integer Linear Programming (MILP) optimization models [2] and solved using mathematical optimization techniques such as the branch and cut algorithm. Today, various metaheuristics are widely used to solve unit commitment problems [3] and are even predicted to become the standard for design optimization problems in the near future [4, 5].
While mathematical optimization is guaranteed to find the global optimum of a feasible problem, the complexity of global optimization problems can become so large that the global optimum cannot be found within a reasonable time. [6, 7] Metaheuristic methods, on the other hand, can solve complicated models in a shorter time frame, but do not guarantee the optimality of the results [8].
Since both solution techniques have their advantages, it is important to have a decision guide when to use which technique. Nevertheless, there is little research on how metaheuristics compare to other solution techniques such as mathematical optimization and heuristics in the field of energy system optimization. This paper aims to make a start in filling this research gap by comparing the performance of two popular metaheuristics, the Speed-constrained Multi-objective Particle Swarm Optimization (SMPSO) [9] and the Non-dominated Sorting Genetic Algorithm II (NSGA II) [10], with the performance of classical mathematical optimization on a MILP model for the design optimization of an energy system. The two selected metaheuristics are based on two different approaches.The NSGA-II is based on the principle of survival of the fittest, i.e. inheritance, mutation and selection of the best genes or, in this case, the decision variables. The SMPSO, on the other hand, is based on the flocking behavior of birds and schools of fish, and takes into account the speed of the neighbor

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as well as a random change in the speed parameter.

There is a significant amount of papers comparing different metaheuristic algorithms [11, 12, 13, 14] but only a little research on how different solving techniques like metaheuristics, heuristics and classical mathematical optimization perform compared to each other, especially regarding the differences in results.
In Suh et al. [15] it is shown that metaheuristics outperform the heuristic layout of professionals by far in terms of finding the global optimum for this case study. Here, the layout refers to building decisions such as window area, insulation thickness and light bulbs.
In Silveira et al. [16] five different metaheuristics are compared to MILP, MINLP and MISOCP with classical optimization. As a case study, three different configurations of distribution systems are considered. The computational time of the metaheuristic optimization was significantly shorter than that of the mathematical optimization. For large problems, metaheuristics even found a better solution with regard to the objective function than classical mathematical optimization.
Stojiljković et al. [6] and Schmeling et al. [17] use a combined approach where the optimization problem is decomposed into two new problems. The main problem is the design optimization including synthesis, which is solved by a metaheuristic. The subproblem is the operational optimization, which is formulated as a MILP and solved by the brunch-and-cut method. The authors did not compare the results with solving the original problem using the branch-and-cut algorithm directly on the design optimization problem. However, it is noteworthy that the proposed approach has the potential for a comparison between metaheuristic algorithms and mathematical optimization, since the same constraints can be used to model the individual components of the power system.
In the following, we will compare the decomposed metaheuristic design optimization with a mathematical design optimization also referred to as structural optimization approach using MILP only.

The remaining sections of this paper are structured as follows: Section 2. provides an overview of the case study used to compare the methods. Section 3. introduces the unit commitment model used for design optimization and the corresponding metaheuristic approach. Section 4. presents and compares the results of the case study. Finally, section 5. discusses the results and draws relevant conclusions.

2. Case Study: Residential building energy system

Decarbonizing the building stock is a major challenge. As most of the existing building heat demand is met by fossil fuel-fired boilers, there is a high demand to find optimal solutions considering alternative technologies that reduce local emissions at minimal cost. Therefore, in this study, a residential building is chosen to test different approaches for designing energy systems as a problem with high practical relevance. A key strategy for decarbonizing building energy systems is to switch from combustion-based towards electricity-based heat supply combined with local electricity generation. Therefore, the design optimization of the energy system in this study considers a gas boiler, an electric heat pump for heat generation combined with heat storage. Local electricity can be provided by a photovoltaic system combined with battery storage. All components can be considered as options for the optimization algorithm. Thus, not all components have to be part of the final solution of the corresponding optimization problem. The full set of technology options and the possible energy flows in the system are given in Figure 1.

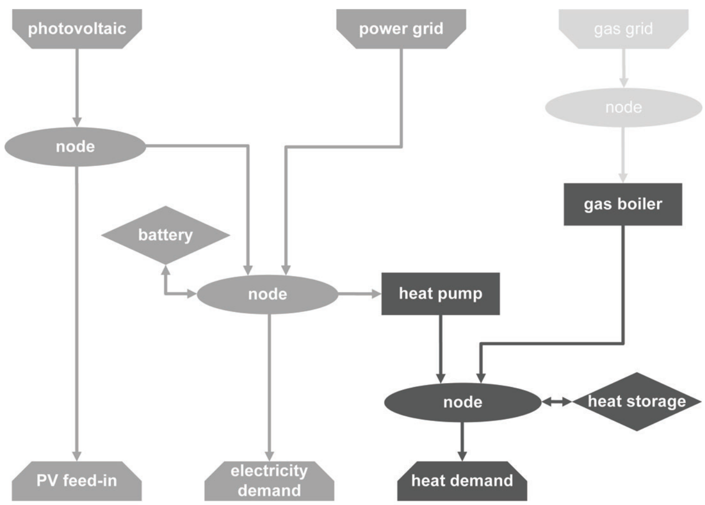

Figure 1: Schematic representation of the residential building energy system

The energy prices used in the model are form 2021. The emission data and the energy prices are listed in Table 1.

Table 1: Energy prices and CO_2 emissions

Data type	Costs	CO_2 Emissions
Electricity	0.337[€/kWh] [18, 19]	478 [g/kWh][20]
Gas	0.083 [€/kWh] [21, 22]	247 [g/kWh] [23]
PV feed-in tariff	0.073 [€/kWh] [24]	-

The investment costs for the various components are listed in Table 2.

Table 2: Investment costs of the components

Component	C_{var}	C_{fix}
Photovoltaics [25]	1260[€/kWp]	258 [€]
Heat pump [20]	426 [€/kW]	7072 [€]
Gasboiler[20]	445 [€/kW]	724 [€]
Heatstorage[26]	1000[€/kWh]	600 [€]
Battery (capacity)[27]	432 [€/kWh]	2130 [€]
Battery (inverter)[28]	150[€/kW]	-

The demands are modeled by using the open source tool *districtgenerator* [29] released by E.ON Energy Research Center, RWTH Aachen which is based on validated models from other research projects. It offers the possibility to define residential districts by specifying archetype buildings and generating building-specific energy demand profiles. The occupancy and corresponding electricity profiles are generated by a stochastic model based on [30]. The occupancy data is used to model the heating demand by modeling the domestic hot water consumption based on [31]. The space heating demand calculations are based on a 5R1C model according to EN ISO 13790 [32], where the corresponding building parameters are given by TABULA archetype buildings [33]. For this study a single building is defined. The *building type* is defined as a multi family house. The *construction year* is set to 1990 and the *retrofit state* according to TABULA [33] is a assumed to be in an usual refurbishment state. The *reference floor area* is defined as 778 m^2. The location is set to Potsdam (Germany) and the corresponding TRY dataset is used for the weather profiles.

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3. Optimization models

In this section, the energy system model is described, starting with the unit commitment model for energy system operation, which forms the basis for both mathematical optimization and metaheuristic approaches. Next, the extensions to the unit commitment model necessary for design optimization are presented. Then, the modifications made to the unit commitment model in order to use the model with the metaheuristic algorithms are summarized. Finally, the metaheuristic method and the optimization setup are described.

3.1. Model for energy system operation

A mixed-integer linear unit commitment model is used as the base model for both the mathematical design optimization and the metaheuristic optimization. The model is implemented using the energy system optimization framework oemof. The household demand for electricity, heat, and hot water are modeled as sinks. The gas and electricity grids are represented as sources. The PV generation profile is simulated using a PVLib [34] model. For all other energy system components, we used the base class Transformer to write our own models. These models are briefly described below:

Let T be the set of all time steps that are considered in the optimization.

All components are limited by a maximum power rating $P_{el/th,max}$, either electric power or heat. Except for the heat pump and the gas boiler, all components are allowed to operate between 0 and this maximum power rating.

$$0 \leq P_{el/th}(t) \leq P_{el/th,max} \text{ for all } t \in T \tag{1}$$

For the heat pump and the gas boiler a minimum part load MPL is set as a percentage of the maximum power rating. To allow the output power to be zero, the binary variable Y_{op} is introduced.

$$Y_{op}(t) \cdot MPL \cdot P_{el/th,max} \leq P_{el/th}(t) \leq P_{el/th,max} \text{ for all } t \in T \tag{2}$$

The COP of the heat pump is modeled as ambient temperature dependent:

$$P_{th}(t) = P_{el}(t) \cdot cop(T(t)) \text{ for all } t \in T. \tag{3}$$

The efficiencies of the boiler and the battery storage are modeled as constant. The storage level is limited by a maximum storage capacity for both the battery and the thermal storage. The battery has no self-discharge, while the thermal storage has both temperature-dependent and level-dependent losses, which are calculated using the volume, the volumetric thermal transmittance and the density of the storage medium. The objective function of the model is to minimize the operating costs of the energy system, consisting of fuel and electricity costs for purchasing gas and electricity from the respective grid. Electricity produced by the PV system that is not consumed but fed back into the grid is compensated by the respective feed-in tariff. CO_2 emissions are calculated using constant emission factors assigned to the consumption of gas and electricity from the grid.

3.2. Design optimization model for the mathematical optimization

To transform the unit commitment model into a design optimization model constraints regarding the sizing of the components are added to the optimization problem. Additionally, the maximum power rating parameter $P_{el/th,max}$ of the unit commitment problem becomes a variable in the design optimization problem. To limit the solution space the maximum power rating is limited by P_{MAX}.

$$0 \leq P_{el/th,max} \leq P_{MAX} \tag{4}$$

In order to limit the solution space and thus the computation time, lower and upper bounds for the parameter P_{MAX} are introduced. The lower bound is set to zero and the upper bounds are determined by analyzing the energy demand, such as the maximum required heat output. For the PV system, the available roof area is used as the limit. When $P_{el/th,max}$ is zero the component is not built.

In both storage models, not only the capacity but also the maximum electrical/thermal power is optimized. For the battery, a power-to-energy ratio is implemented, to ensure a realistic battery layout. Furthermore, the total charged energy is limited according to cycle and calendar lifetimes.

$$\sum_{t \in T} P_{in}(t) \leq Capacity \cdot \frac{Lifetime_{cycle}}{Lifetime_{calender}} \tag{5}$$

We perform a multi-objective optimization with two objectives, the total annual cost of the system and the annual CO_2 emissions, to compute a Pareto front. For this purpose, the energy system is first optimized with respect to costs only. Here the objective function is the systems total annual costs (*TOTEX*).

$$TOTEX_{annual} = CAPEX_{annual} + OPEX_{annual}. \tag{6}$$

with $OPEX_{annual}$ the grid energy costs and maintenance costs for one year and $CAPEX_{annual}$ the investment costs annualized over the lifetime of the technology. Since the relation between the size of a component and its price is not linear, the *CAPEX* is calculated using a fixed C_{fix} and a variable C_{var} price component.

$$CAPEX = C_{fix} + P_{el/th,max} \cdot C_{var} \tag{7}$$

To build the Pareto front, an epsilon constraint is added to the optimization problem. This constraint limits the total annual CO_2 emissions of the system to x percent of the CO_2 emitted in the cost optimal case.

$$CO2_{total,annual} \leq CO2_{total,annual,costoptimal} \cdot \frac{x}{100} \tag{8}$$

In our case, x is decreased in steps of 2.5 starting at 100 (the cost optimal case) and stopping at 50 (a near CO_2 optimal case). No exclusively CO_2 optimization is carried out.

3.3. Metaheuristic optimization

The optimization problem has been decomposed into two subproblems, as proposed in [17]. The selection and sizing of the technologies here referred to as metaheuristic design optimization and the operational optimization of the resulting energy system (cf. Figure 2). The design optimization passes a set of variables to the operational optimization which returns the annual costs and emissions for each set of variables. Afterwards, the metaheuristic design optimization selects a new set of variables according to the KPIs, here CO_2 emissions and TOTEX, and passes it back to the operation optimization. The process ends when the termination criterion, here the computation time limit, is reached.

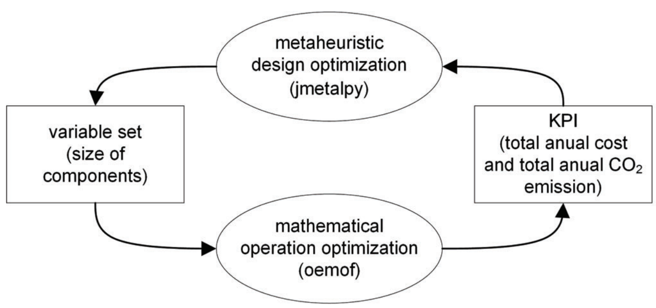

Figure 2: Two-level design optimization

The operation optimization is performed with the same unit commitment model described in 3.1.. The MILP is solved with Gurobi using a branch-and-cut algorithm. The emissions during the operation are limited by an emission factor to achieve comparable results to the MILP design optimization, which indirectly optimizes the operation to meet the global emission limit.

The main problem of design optimization is solved by a metaheuristic algorithm. The algorithm can determine the dimensions and capacity of the technologies and limit the emissions of the operation by an emission factor. The upper and lower bounds are set exactly as described in 3.2.

Analogous to mathematical optimization, two objective functions are used for metaheuristic optimization: The total annual costs and the total annual CO_2 emissions.

The metaheuristic optimization is done with the python package jmetalpy which uses the java-based framework jmetal.

3.4. Optimization Setup

In this section we give some insight into our setup and the software and settings we used. The calculations were performed on a computer with an Intel(R) Xeon(R) W-1390P @ 3.50Ghz and 128 GB RAM.

The design optimization MILP is solved with Gurobi Optimizer version 9.1.1 using up to 16 threads. A mipgap of 0.01 and a time limit of 6 hours per optimization is used. If the time limit is exceeded, the best solution and

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the resulting mipgap are saved and the epsilon constraint method is continued.
The operation optimization MILP is solved with the same solver and solver settings except for the time limit, which is set to 5 minutes. The metaheuristic design optimization uses the default settings of the respective algorithms as presented in jmetalpy. In contrast to mathematical optimization, in metaheuristic optimization, it is not clear whether the global optimum has been found. Therefore, a termination criterion is needed. In order to be able to compare the results of the two methods, a time limit is set as the termination criterion. To make the results of the metaheuristic optimization reproducible, the same random seed is used for all algorithms and runs. All optimizations are computed for a whole year with a time resolution of one hour, i.e. 8760 time steps.

4. Results and Comparison

In this section, we analyze the results of the case study. We begin by examining the metadata of the algorithms used, followed by a comparison of the Pareto fronts. We then take a closer look at the actual energy systems built for the Pareto solutions.

Table 3 summarizes metadata such as the number of non-dominated solutions and the total computation time. A time limit of 20.15 hours and 5 hours was implemented for the metaheuristic methods, as well as a variation of the population or swarm size of 10 and 100. It can be seen that the termination criterion was not exactly met. Both NSGA-II and SMPSO exceeded the time limit. The population and swarm size appear to have a direct influence on the time limit violation. It seems that the termination criterion is applied only after the total generation has been computed. Mathematical optimization using the epsilon constraint produced 19 non-dominated solutions and two dominated solutions. The observed phenomenon is a direct consequence of the results reported in the recent research [35], which concluded that the epsilon constraint approach fails to accurately compute the true Pareto front. As a result, the use of lexicographic optimization is proposed to compute the true Pareto front. The metaheuristic methods found significantly more non-dominated solutions and total feasible solutions than mathematical optimization with the 20-hour time limit. Reducing the population/swarm size resulted in more optimizations and feasible solutions for both algorithms. For NSGA-II, the number of non-dominated solutions also increased significantly. For SMPSO, the number of non-dominated solutions decreased from 48 to 46.

Table 3: Meta data of the optimization

Method	MILP	NSGA-II	NSGA-II	NSGA-II	SMPSO	SMPSO	SMPSO
time limit [h]	N/A	20.15	5	20.15	20.15	5	20.15
total time [h]	20.15	21.40	5.40	20.19	21.66	5.55	20.18
population/swarm size [-]	N/A	100	100	10	100	100	10
generations [n]	N/A	12	3	140	10	3	121
total optimizations [n]	22	1200	300	1400	1000	300	1210
time limit reached [n]	0	11	5	1	11	5	10
infeasible [n]	1	259	91	226	279	94	470
feasible [n]	21	930	204	1173	710	201	730
non-dominated solutions [n]	19	38	14	85	48	22	46

Figure 3 shows the Pareto front of the different design optimization methods. The mathematical optimization Pareto front, hereafter referred to as the MILP Pareto front, is used as a reference in all subsequent figures. The computation time of the methods in this figure is about 20.15 hours (see Table 3) and the population and swarm size is set to 100. The solutions of the mathematical optimization for the case study lead to the best Pareto front in both dimensions. This implies that the Pareto front contains the most cost-optimal and CO_2 efficient Pareto points, in terms of all three algorithms. In this example, we observe that the SMPSO algorithm outperforms the NSGA-II algorithm in terms of finding the cost-optimal solution. However, we found that the NSGA-II algorithm was closer to the MILP front at the bending point of the Pareto curve. In other words, NSGA-II comes closer to the ideal point in Figure 3 than SMPSO. On the other hand, the emission-optimal point found by the SMPSO algorithm is more cost-optimal and has lower emissions than the one found by the NSGA-II algorithm. The emission-optimal solutions found by the metaheuristic methods are much more expensive than the emission-optimal solution of the MILP solution. In addition, the metaheuristic Pareto fronts continue to separate from the MILP Pareto front as emissions decrease.

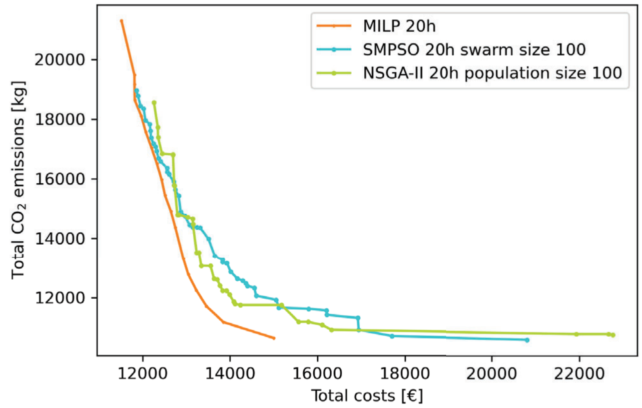

Figure 3: Pareto MILP, NSGA-II and SMPSO (20h time limit and population/swarm size of 100)

In Figure 4, the Pareto front of the MILP mathematical optimization from Figure 3 with a computation time of 20 hours is compared to the metaheuristic methods with a computation time of about 5 hours. The metaheuristic solutions are now much further away from the MILP Pareto front. For the same CO_2 emissions, the metaheuristics incur significantly higher total annual costs than the MILP Pareto solutions. This effect increases significantly with decreasing CO_2 emissions. The SMPSO, in contrast to the NSGA-II, found a point particularly close to the MILP Pareto front. Compared to the NSGA-II, the SMPSO found the solution with the lowest emissions and the solution with the lowest total annual cost.

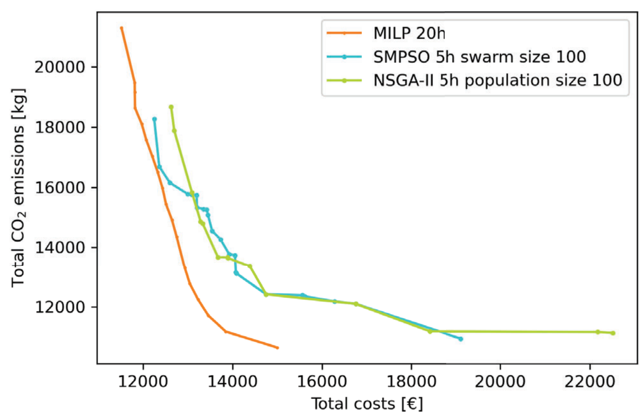

Figure 4: Pareto MILP, NSGA-II and SMPSO (5h time limit and and population/swarm size of 100)

In Figure 5 the population/swarm parameter was set to 10. The time limit here is 20.15 hours, as in Figure 3. The Pareto fronts of the NSGA-II and the SMPSO are close. The Pareto front of NSGA-II covers a smaller solution space than that of SMPSO. Compared to Figure 4, the Pareto front of the metaheuristics is much smoother. The NSGA-II and SMPSO perform slightly better with a swarm/population size of 10 in the emission optimal range. This could be due to the significantly higher number of generations. Compared to Figure 3, the algorithms perform better in some parts of the Pareto front and worse in others.

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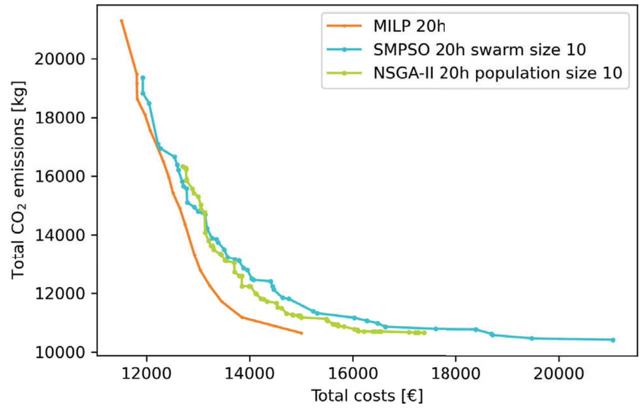

Figure 5: Pareto MILP, NSGA-II and SMPSO (20h time limit and population/swarm size of 10)

Closer examination of the energy systems built for the Pareto optimal solutions for the 20h mathematical optimization and the 20h metaheuristics with a population/swarm size of 100 reveals significant differences between the mathematical optimization and the metaheuristics. Figure 6 show the components and their sizes for all Pareto optimal results. The plots are sorted from cost optimality on the left to CO_2 optimality on the right .

Regarding the mathematical optimization, the capacity of the heat producers increases moderately across all solutions, as shown in 6a. It is noteworthy that only in the cost-optimal case no heat pump is built. For the next Pareto point, the gas boiler capacity decreases slightly and a heat pump is installed. Interestingly, the boiler and heat pump capacities remain almost constant for ten Pareto points until a tipping point is reached where no gas boiler is installed and the heat pump provides the whole heat demand.

Conversely, for both metaheuristics, a gas boiler is built for all Pareto optimal solutions.

For the NSGA-II algorithm, the gas boiler capacity initially decreases as emissions decrease, but then increases significantly for the last eight CO_2 optimal solutions, so that the gas boiler capacity for the cost-optimal and emission-optimal cases are nearly equal.

On the other hand, for the SMPSO algorithm, the gas boiler size remains nearly constant for the more cost-optimal Pareto points until a tipping point where only a very small gas boiler is built. But similar to the NSGA-II algorithm, the gas boiler size increases again for more emission-optimal solutions.

When analyzing the operation of the boiler over the course of a year for the SMPSO and NSGA-II algorithms for the emission-optimal solution, it can be observed that the boiler was rarely used, running only 54 hours and 249 hours, respectively, throughout the year. The installation of the boiler despite its limited use can be attributed to the absence of indirect emissions which are not included in this anaylsis. In fact, there are no "investment emissions" for the components. Similar to the mathematical optimization, figures 6b and 6c show that the heat pump size increases towards the CO_2 optimal solutions. However, in contrast to the mathematical optimization, the overall installed capacity of heat producers is higher for the metaheuristic algorithms.

Regarding the installed photovoltaic capacity, a constant growth along the Pareto front towards the CO_2 optimal solution is observed for both the mathematical optimization and the metaheuristics.

Despite an overall increase in storage capacity towards the CO_2 optimal solution for all three algorithms, significant differences in storage capacity are observed between the mathematical optimization and the metaheuristics.

The most notable difference is the approximately twofold increase in total storage capacity for the metaheuristics compared to the mathematical optimization. In the mathematical optimization, only the thermal storage is built in the cost-optimal case, and its size increases along the Pareto front until an additional battery is installed. At this juncture, the size of the heat storage drops before increasing again. Furthermore, the size of the battery increases steadily until the point where no gas boiler is installed. At this point, the size of the battery decreases to zero before increasing again toward the emission-optimal solution.

Similarly, both metaheuristics build thermal storage for all Pareto points. The NSGA II algorithm maintains a relatively constant heat storage capacity for most of the Pareto front. However, for more CO_2 optimal cases, the heat storage doubles in size. The battery size increases moderately toward the CO_2 optimal solution until a tipping point where the battery capacity increases dramatically to over 230 kWh.

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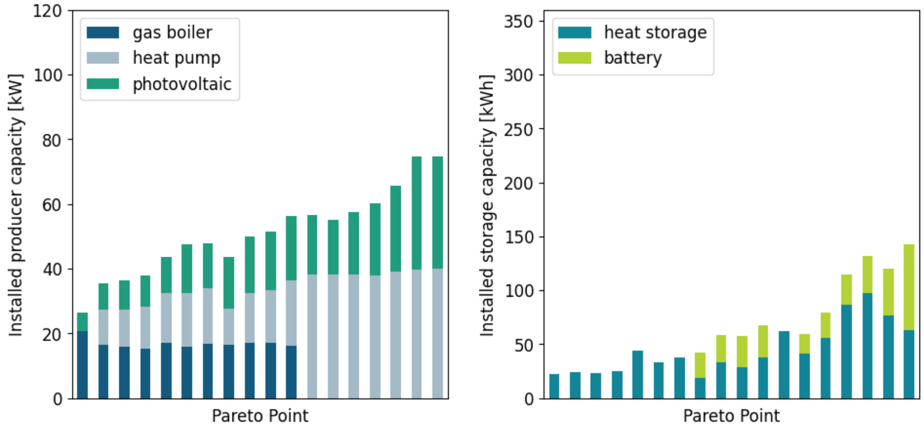
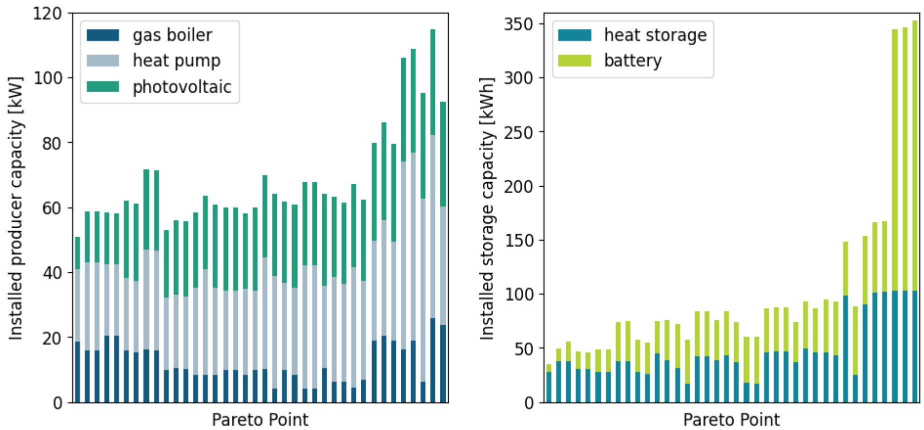
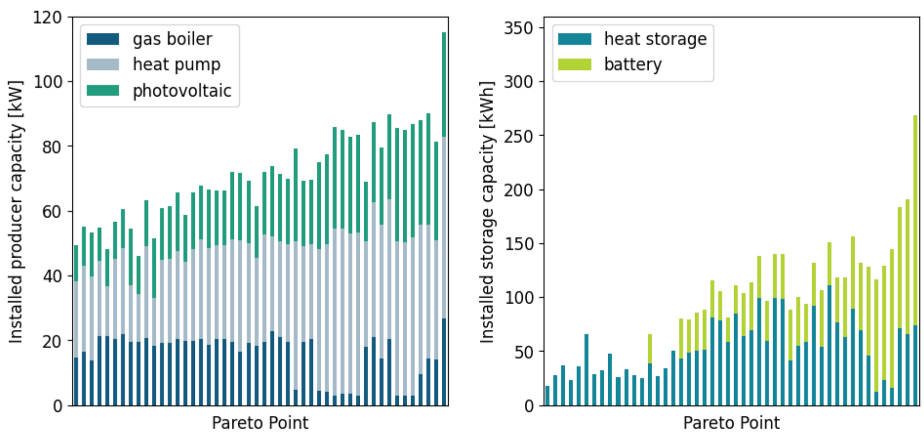


(a) mathematical optimization (MILP)



(b) metaheuristic optimization (NSGA-II)



(c) metaheuristic optimization (SMPSO)

Figure 6: Energy systems from the pareto optimal results for different algorithms

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In contrast to the NSGA-II algorithm, the heat storage capacity fluctuates significantly throughout the Pareto front of the SMPSO algorithm, peaking in the middle. In addition, the SMPSO algorithm does not install a battery until almost the middle of the Pareto front. Then the battery capacity increases steadily.

The higher costs associated with the emission-optimal solutions generated by the metaheuristics, as observed in Figures 3-5, can be attributed to the substantially higher storage and producer capacities employed by these algorithms. The reason for this could be that metaheuristics are severely penalized for choosing infeasible solutions or reaching the time limit in operations optimization. Therefore, the trained behavior of these metaheuristics is biased toward quickly producing feasible solutions, which results in larger capacities. This is probably the cause of the unexpected behavior of the metaheuristics in constructing a boiler for all solutions. This behavior can be reduced by allowing a higher time limit for operations optimization and more time overall for the computations, so that the metaheuristics can compute more generations.

Another notable characteristic of metaheuristics is the greater fluctuation in capacity sizing, as opposed to the smoother trend observed in mathematical optimization. This fluctuation can be explained by the random nature of the metaheuristics. This effect should be further investigated in future studies.

5. Conclusion

This paper compares two methods and their results for the design optimization of a residential building. A multiobjective mathematical design optimization approach and a two-stage metaheuristic design optimization approach using the metaheuristics SMPSO and NSGA-II were described, and the Pareto fronts and energy system designs of the Pareto solutions generated by each algorithm were analyzed.

In our case study, the mathematical optimization resulted in the best Pareto front. However, it should be noted that the calculation time was quite extensive at 20 hours. To reduce the computation time of the mathematical design optimization, a larger step size of the epsilon constraint could be applied. This would reduce the number of optimizations but at the same time the resolution of the Parteo front. Additionally, a lexicographic optimization approach can be used to compute the real Pareto front, which could lead to more accurate and efficient results and avoids caluating dominated solutions. Further research is needed to fully explore these possibilities and improve the efficiency of the optimization process.

Furthermore, our study has shown that the choice of parameters such as swarm/population size has a strong influence on the optimality of the results. Depending on the choice of metaheuristic parameters, either the NSGA-II or the SMPSO performed better. Therefore our optimization results do not allow a conclusion which of the two metaheuristic algorithms is better suited for the design optimization of residential buildings. In addition, it was shown that it is possible to compute a Pareto front with the metaheuristic approaches in shorter time frames, but this results in a significantly worse Pareto front compared to the MILP and 20 h metaheuristic solutions. For this reasons, further studies are necessary.

As mentioned above, the parameter settings of metaheuristic algorithms have a large influence on the quality of the results. For this reason, guidelines for the parameterization of metaheuristic design optimizations for energy systems would be beneficial. In order to ensure the comparability of the results of the different methods, a time limit was used as a stopping criterion in this study. However, when metaheruistic methods are used to obtain results in practice, a termination criterion is needed that allows a statement to be made about the quality of the solution or the convergence of the algorithm. Since one of the main advantages of metaheuristics is their computational efficiency compared to mathematical optimization, it would be worthwhile to investigate how solution-quality-oriented termination criteria such as the hypervolume could enhance performance. In addition, a warm start, i.e. the implementation of coherent start variables such as a standard desgin according to DIN norms, could make the solution process more efficient and robust. The factor that metaheuristics are not limited to MILP problems, like mathematical optimization, but can also be used with Non Linear Programming or simulation was not considered in this study. Hence it would be interesting to investigate a single-stage metaheuristic optimization instead of the two-stage approach used in this paper.

Lastly, additional case studies involving more complex energy systems and a wider range of scenarios are required to develop a reliable guideline for selecting an appropriate design optimization method.

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