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Operation planning with thermal storage units using MILP: Comparison of heuristics for approximating non-linear operating behavior

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Abstract:

For operation planning in industrial energy systems mixed integer linear programming (MILP) is the go-to method because of its reliability and the huge advances in MILP algorithms in recent years. MILP is especially well suited for planning the use of storage units, even if including the non-linear operating behavior of thermal storages is still a big challenge – especially if partial load cycles are considered. To model the storage behavior, a multi-variate non-linear function has to be linearized and incorporated into the MILP model. The key for good performance in MILP is using as few linear pieces as possible to achieve the required accuracy. We consider two types of piecewise-linear models: triangulation on a grid and general triangulation.

In this paper, we present different heuristics for computing efficient piecewise-linear approximations of non-linear functions. As a use case we consider the behavior of a thermal storage unit. We apply the heuristics to compute piecewise-linear approximation of the non-linear operating behavior and discuss the results. We then compare the performance of the models in a MILP model for the operation planning of an energy system. For translating the piecewise-linear function to MILP we consider state-of-the-art approaches with a logarithmic number of binary variables.

Our results show that gridded triangulation models in combination with logarithmic MILP formulations can be used for data-driven modeling of non-linear operating behavior of devices. We highlight the potential of this approach for realizing adaptable operation optimization of energy systems.

Keywords:

Thermal energy storage, Packed bed reactor, Unit commitment, MILP, Data-driven modeling.

1. Introduction

In an effort to curb climate change governments are imposing increasingly strict emission targets on industry. The recent increase in energy prices puts additional pressure on companies. Companies, especially in the energy-intensive industries, are forced to reduce primary energy consumption and increase energy efficiency. To do this, companies will have to employ a wide range of different solutions. Re-using excess heat on site, i.e. reducing waste heat, is getting more and more important.

The total global waste heat potential in 2016 was estimated to be more than 68 TW h [1]. Large parts of it can be attributed to the industrial and power generation sectors. In many cases, excess heat remains unused because heat demand occurs at another time than waste heat is available. Thermal energy storage (TES) systems can store a considerable share of the available waste heat and make it available. In this way, TES not only increase the flexibility of energy-intensive industrial processes [2, 3] but they also help increasing the overall energy efficiency and reducing green house gas emissions.

To operate the storage efficiently a lot of planning is required, especially for non-periodic processes. For operation planning in industrial energy systems unit commitment (UC) methods based on mixed integer linear programming (MILP) are the go-to methods because of their reliability [4]. The huge advances in MILP algorithms in recent years allows it to solve UC problems very quickly [5]. MILP has the advantage (compared to genetic algorithms and heuristic storage management techniques) that it reliably finds the optimal storage trajectory because, as a deterministic optimization method, it always converges to the global optimum. UC can be used to organize energy supply in a cost-optimal manner and to ensure that energy demand is always met.

For the present study, we consider a packed bed regenerator (PBR) that is to be used for waste heat recovery from the off-gas in a steel production process [6]. Because of the harsh operating conditions, slag dust is

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expected to accumulate inside the storage over time, which will change the operating behavior. In an ongoing research project a digital twin platform is being developed to monitor the PBR and to operate the storage efficiently and reliably during its whole lifetime [7]. The digital twin platform features a thermal model of the PBR that is adapted regularly to model the operating behavior of the storage accurately at all times. This thermal model can be used to predict the behavior of the storage, but it cannot be integrated into the UC model.

To model the behavior of the PBR in the UC model accurately, the dependence of the maximum charging and discharging power on the state of charge has to be considered. The maximum power diminishes, when the storage is almost fully charged or discharged, because the power depends on the position of the thermocline inside the packed bed [4]. When the storage operates in partial charging and discharging cycles (which will usually be the case), the state of charge at the time of switching also has to be considered [4].

A big challenge with modeling the PBR for the UC is automatically deriving an accurate model for its non-linear operating behavior. The model, which was proposed in [4], was set up by hand. For this reason, it cannot be used for the automated model adaption which we intend to realize on our digital twin platform.

In this paper, we discuss different methods for modeling the storage behavior in the UC model and evaluate their performance both in terms of how well they approximate the operating behavior and how they affect the solving time of the UC problem. We show that gridded models are viable for incorporating data driven models in MILP. With this approach, we can adapt the UC model to the changing operating behavior of the PBR and thus realize reliable and accurate operation planning for an energy system with a TES over its whole lifetime.

2. Methods

2.1. Modeling non-linear operation characteristics in MILP

As the name suggests, mixed integer linear programming (MILP) is limited to linear relations between continuous and integer variables. To include non-linear characteristics, they have to be approximated by piecewise-linear functions. These piecewise linear functions are then incorporated into the MILP model by using auxiliary binary variables to distinguish between the individual pieces of the piecewise-linear function.

The ability to (approximately) include non-linear behavior in MILP comes at a price: Since the complexity of MILP problems increases exponentially with the number of binary variables, detailed piecewise-linear functions can severely affect the performance.

Piecewise linear-functions can be incorporated into a MILP problem with different formulations. For functions with one independent variable, the formulation is known as Special Ordered Set of type 2 (SOS2). For functions with more than one independent variable (as in our use case), different formulations were suggested [8,9] which differ in the number of auxiliary binary variables that are required to distinguish the simplices (pieces).

The direct extension of the concepts of SOS2 formulations to higher dimensional functions is known as Convex Combination (CC) [8]. This formulation can be used for any piecewise-linear function that uses simplices and the required number of auxilary binary variables increases linearly with the number of simplices. When the simplices are arranged in a rectangular grid, a more efficient formulation (log CC) can be used, where the number of auxilary binary variables increases only logarithmically with the number of simplices [9]. For piecewise-linear functions with many simplices, this can result in considerable savings.

2.2. Approximating storage behavior for MILP

In a packed bed regenerator, the maximum charging and discharging power depends on the state of charge and and on the initial state of charge of the charging/discharging cycle [4]. This behavior can be described by a non-linear function with two independent variables (state of charge and initial state of charge). To model the storage behavior in a UC problem in MILP, this non-linear function has to be approximated by a piecewise-linear function.

We assume that the non-linear function is given by a data-set (e.g. simulation data from a thermal finite volume model). A general piecewise-linear model with two independent variables is a triangulation of the domain with function values at each vertex. The goal of the approximation is, thus, to determine the optimal triangulation and to estimate the values at the vertices. For this problem, hardly any solutions have been proposed in literature.

From a MILP perspective gridded models have the advantage of more efficient MILP formulations, but with free triangulations (i.e. triangulations that are not confined to a grid) the target accuracy could potentially be achieved with much less simplices. Which modeling approach is more viable thus boils down to how many simplices are needed to achieve the target accuracy and by how much the solving time of the UC problem can be accelerated by using a logarithmic formulation.

For approximating a two dimensional function with a (non-gridded) triangulation, a heuristic was proposed [10]. Starting from an initial triangulation of the domain, the simplices are split according to a specific strategy, until

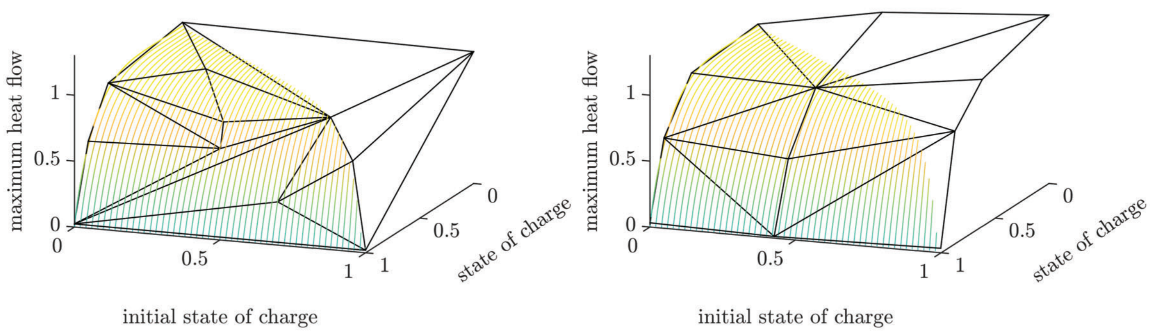

Figure 1: Illustration of the linearization of the maximum charging power with normalized axes. Free triangulation (left) and gridded triangulation (right), both with a root mean square error of $2 \cdot 10^{-2}$.

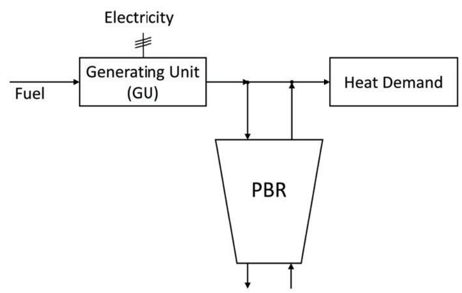

Figure 2: Illustration of the energy system for the unit commitment. Adapted from [4]

the required accuracy is achieved. We found that results with this method were barely satisfactory. Also, this kind of triangulation does not allow for the most efficient logarithmic formulation.

For this reason, we developed our own approximation algorithm for models that use a gridded triangulation. Determining the optimal grid positions and function values at the vertices is posed as a non-linear least-squares problem. As long as the target accuracy is not reached, additional grid lines are added with a heuristic and the optimization is repeated. To improve the convergence, gradients of the non-linear optimization problem are computed analytically. Gradient and grid position regularization is used to improve the quality of the model. A detailed description of the algorithm is beyond the scope of this paper but may be the content of a future publication.

Figure 1 shows an illustration of the piecewise-linear approximation of the maximum charging power of the PBR with each heuristic. In both cases, the target accuracy in terms of the root mean square error (RMSE) was $2 \cdot 10^{-2}$. The model using a free triangulation needed 16 simplices while the one with the gridded triangulation needed only 12. Unfortunately, the heuristic for fitting non-gridded triangulations cannot consistently exploit the additional degrees of freedom and compute better approximations with fewer simplices.

2.3. Unit commitment model

For evaluating the performance of each modeling approach and the effect of the logarithmic formulation, we used the same UC model as Koller et al. [4]. In this UC model, a very simple energy system (see Figure 2) consisting of a generating unit (GU), the PBR as a heat storage and a heat demand is considered. The heat demand of the process has to be covered at all times. The GU produces both heat and electricity. Electricity is sold at the electricity market at a fluctuating but known price. To allow the GU to shut down during times of low electricity prices, the PBR is used to store heat and supply the demand.

The UC model has a prediction horizon of 8 days with time steps of 1 hour. This results in 192 time steps. A detailed documentation of the model and its parameters can be found in [4].

The output of the UC problem is illustrated in Figure 3. The diagram on the top shows how the heat demand is covered by either a heat flow from the GU or from the PBR. The fluctuating electricity price that affects the optimal operation of the GU is also shown. In the bottom diagram, the operating trajectory of the PBR is shown.

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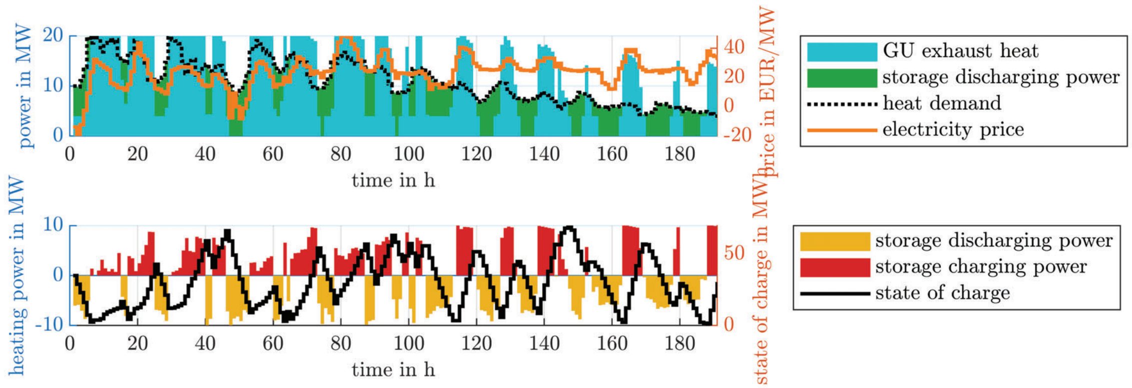

Figure 3: Illustration of the output of the unit commitment problem.

Table 1: Number of simplices, number of auxilary binary variables and resulting solving time in MILP for each model type and MILP formulation.

	RMSE	simplices	auxilary binary variables	solving time in s
free, CC	$1 \cdot 10^{-1}$	10	10	17
	$5 \cdot 10^{-2}$	10	10	18
	$2 \cdot 10^{-2}$	32	32	160
	$1 \cdot 10^{-2}$	64	64	7423
grid, CC	$1 \cdot 10^{-1}$	16	16	49
	$5 \cdot 10^{-2}$	16	16	49
	$2 \cdot 10^{-2}$	24	24	72
	$1 \cdot 10^{-2}$	60	60	1651
grid, log CC	$1 \cdot 10^{-1}$	16	6	5
	$5 \cdot 10^{-2}$	16	6	5
	$2 \cdot 10^{-2}$	24	8	28
	$1 \cdot 10^{-2}$	60	12	54

The bars illustrate heat flow to and from the storage. The line shows the state of charge.

3. Results and discussion

The aim of this study is to determine the most efficient way to derive data-driven models for the operating behavior of a PBR for use in a UC problem.

We consider two types of piecewise-linear models: free triangulation and gridded triangulation. Both types of models can be included with the MILP formulation known as Convex Combination (CC). The gridded model also allows for a logarithmic formulation (log CC). Which approach is more efficient depends 1) on the number of simplices that each modeling heuristic requires to reach the target accuracy and 2) on the speed-up due to the reduced number of auxiliary variables with the log CC formulation.

We used each fitting heuristic with the target accuracies, in terms of the root mean squared error (RMSE), of $1 \cdot 10^{-1}, 5 \cdot 10^{-2}, 1 \cdot 10^{-2}$. The results are aggregated in Table 1. To approximate both the maximum charging and discharging power at an accuracy of $1 \cdot 10^{-1}$ and $5 \cdot 10^{-2}$, 10 simplices were required with the heuristic using a free triangulation, while the heuristic using a gridded triangulation needed 16 simplices. At higher accuracies, the heuristic with free triangulation needed a few more simplices than its gridded couterpart. It could not leverage the additional degrees of freedom to achieve the target accuracy with fewer simplices.

The fourth column in Table 1 shows the number of auxiliary binary variables that is required to include the piecewise-linear function in the UC problem. The number of binary variables can be expected to have a huge impact on the solving time since 1) the complexity of MILP problems increases exponentially with the number of binary variables and 2) the models describing the operating behavior of a device in UC have to be replicated at every timestep. Since our UC problem has 192 timesteps, a model that requires 10 auxiliary binary variables introduces 1920 additional binary variables into the MILP problem.

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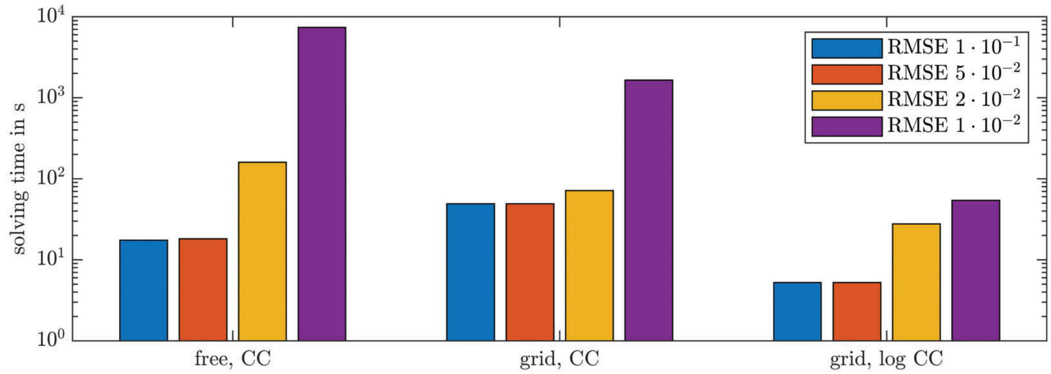

Figure 4: MILP solving time with different models for the operating behavior of the thermal energy storage.

The UC problem was formulated using YALMIP R20210331 [11] in Matlab R2022a. The problems were solved using Gurobi 10.0.0 on a 128-core system (AMD EPYC 7702P) with 256 GB RAM. The results are shown in the last column in Table 1 and are illustrated in Figure 4.

The exponential increase of solving time with the increasing number of simplices and consequently auxiliary binary variables is clearly visible (note the logarithmic scaling of the y-axis). The heuristic using a free triangulation performs slightly better than its gridded counterpart at low accuracies where it managed to make due with fewer simplices. Nevertheless even there it is outperformed by the gridded model using the logarithmic formulation.

4. Conclusion

In this paper we compared two heuristics for deriving data-driven models of the operating behavior of a thermal storage unit in a unit commitment problem. This data-driven modeling approach will be applied in a model adaption service of a digital twin that provides operation planning functionality. The goal is to manage the storage optimally taking into account the degradation of the charging/discharging performance due to harsh operating conditions.

The first heuristic used a free (i.e. non-gridded) triangulation to approximate the non-linear storage behavior, while the other used a grided triangulation. Even if — in theory — the free heuristic should be capable of achieving the target accuracy with fewer simplices than its gridded counterpart, it only did so at low accuracies. Since much more efficient MILP formulations can be used to incorporate gridded models into the unit commitment problem, this model type showed the best performance across the board.

Our results demonstrate that gridded models in combination with the logarithmic MILP formulation are well suited for deriving data-driven models of non-linear functions with multiple independent variables for MILP problems. This allows us to realize adaptable operation optimization on a digital twin platform. Research is already underway to test our approach in a lab test environment and evaluate its potential in a steel mill.

Another interesting direction for future research would be to improve the heuristics for deriving models with a free triangulation. If the additional degrees of freedom can be exploited, this would lead to very efficient models especially if no high accuracy is required or if the function has features that are incompatible with the main grid directions.

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