New approach for a general expression of effectiveness

***Malick Kane**[a]**, Daniel Favrat**[b]*

[a] *HES-SO//FR, University of Applied Science in Fribourg, Mechanical Engineering Department (ENERGY institute), Fribourg, Switzerland, malick.kane@hefr.ch,* ***CA***
[b] *EPFL, Ecole Polytechnique Fédérale de Lausanne, Switzerland, daniel.favrat@epfl.ch*

Abstract:

Providing energy services can be achieved by various technologies or combinations of technologies and it is important to be able to characterize the quality of these different options. In practice, different indicators of the quality of energy processes can be defined: the effectiveness based only on the First Law of thermodynamics or the exergy efficiency based on the exergy balance, thus accounting for both the First and the Second Laws of thermodynamics. While the exergy efficiency definition is general and applies to all systems with values always ≤1, it is not the case of the effectiveness. Today, there is no general definition and formulation of effectiveness that can be applied to all technologies and/or energy conversion systems. The most general formulation of effectiveness given in the literature is indeed mainly valid for processes that take place above the atmospheric temperature. It must however be adapted for refrigeration systems and also for any conversion systems providing simultaneously energy services for both heating and cooling. In this article, we propose a new approach for a general expression of the effectiveness suitable for any energy conversion systems including heat pumps and cogeneration systems with combined heating and cooling. The main advantage of such a method is therefore to provide simple and generic expressions of global exergy losses and efficiencies of any systems in relation to the conventional performance indicator (the effectiveness) most commonly used by engineers to estimate energy losses of a system.

Keywords:

Thermodynamics; Exergy losses; Exergy efficiency; Effectiveness; Explicit relations of exergy

1. Introduction

Energy services can be provided by a variety of types of processes, technologies or combinations of technologies, including boilers, powerplants and heat pump/refrigeration systems with or without cogeneration(combining power, heating and/or cooling). The energy processes can take place in various forms, accompanied by different losses: internal and external losses. The internal losses are resulting from various irreversibilities (dissipations, thermal devaluation and others) while the external losses are related to the energy released to the atmosphere (heat losses, exhaust losses, energy evacuated by a cooling media). In practice, different indicators of the quality of energy processes can be defined [1,2]: the effectiveness based only on the First Law of thermodynamics or the exergy efficiency based on the exergy balance, thus accounting for both the First and the Second Laws of thermodynamics. Different approaches, formalisms and nomenclatures of the exergy analysis to identify losses in a system are reported in [3]. The exergy method applied by some authors [4, 5] consists of quantitatively evaluating the global exergy losses $\dot{L}$ on the basis of internal exergy losses $\dot{L}_D$ called also exergy destruction inside the strictly defined system and external exergy losses $\dot{L}_E$ or exergy destroyed between the system and the atmosphere and calculating the overall exergy efficiency. A general approach of exergy formulation has also been proposed in [1, 2] on the basis of an exergy balance carried out on the boundary of the system extended to the atmosphere in order to internalize and attribute all losses to the system and also by subdividing $\dot{L}$ in two subcategories: the dissipation exergy losses inside the system and the heat transfer exergy losses. Based on this approach, the performance of a system can be determined using a general formulation of efficiency by identifying all exergy services (work, heat and flow exergies) received or provided by the system. All these approaches or methods are equivalent in term of using entropy to estimate the global exergy losses and are still not frequently used in industrial sectors, as it may seem too theoretical. Besides, its application, although ideal for comparing different technologies and locating losses, is not easy for practitioners for different reasons: the concept of entropy and the potential maximum of work associated to a flow-energy (co-enthalpy) are not well understood [1].

The effectiveness (based on the First Law of thermodynamics) is most commonly used by partitioners because of the simplicity of using the energy balance equation to identify external energy losses without any knowledge of entropy. It is however not applicable to compare different technologies because of the multiple definitions that exist (engine efficiency, efficiency based on Lower Heating Value or on Higher Heating Value, heating coefficient of performance, cooling coefficient of performance). Today, there is no general definition and formulation of effectiveness that can be applied to all technologies. The most general formulation of effectiveness given in [5] is indeed only valid for processes that take place above the atmospheric temperature. It must therefore be adapted for refrigeration systems but also for any conversion systems providing simultaneously energy services for both heating and cooling.

https://doi.org/10.52202/069564-0012

In this article, we propose a **new approach for a general expression of the effectiveness** applied to all technologies including heat pumps when used in cogeneration with combined heating and cooling. This new approach facilitates the use of exergy theory in a way to highlight, with explicit equations, the existing link and relationship between energy and exergy losses of any system. Results of using such a method are shown for different examples of cogeneration systems with integrated technologies but also for particular basic components like boilers, heat exchangers and air-coolers. The overall exergy losses and efficiencies explicitly are expressed in function of the effectiveness and the energy services balance which can be determined in any case without any knowledge of the entropy. This can be useful for practitioners to determine and know the exergy performance of any energy system based solely on energy balance terms and the effectiveness of the system.

2. Commonly used indicators of the quality of energy processes

In practice, two different indicators of the quality of energy processes can be defined: the effectiveness (ε: so-called "thermal efficiency" or "coefficient of performance") based only on the First Law of thermodynamics or, better, the exergy efficiency (η) based on the exergy balance, thus accounting for both the First and the Second Laws of thermodynamics. The basic idea of the exergy performance indicator given in [1,2] is to use:

$$\eta = \frac{\sum_t \left[\dot{E}_{u,t}^-\right]}{\sum_t \left[\dot{E}_{u,t}^+\right]} \tag{1}$$

Where $\sum_t \left[\dot{E}_{u,i}^-\right]$ represents the total exergy services provided or delivered by the system to consumers in the form of work ($\dot{E}_w^-$), heat ($\dot{E}_q^-$) and transformation ($\dot{E}_y^-$) exergies and $\sum_t \left[\dot{E}_{u,t}^+\right]$ represents the total exergy services received by the system from utilities in the form of work ($\dot{E}_w^+$), heat ($\dot{E}_q^+$) and transformation ($\dot{E}_y^+$) exergies. All terms here are numerically positive. Considering the global exergy loss ($\dot{L}$) in the system, the ***exergy efficiency*** can then simply be formulated as follows:

$$\eta = \frac{\dot{E}_w^- + \dot{E}_q^- + \dot{E}_y^-}{\dot{E}_w^+ + \dot{E}_q^+ + \dot{E}_y^+} = 1 - \frac{\dot{L}}{\sum_t \left[\dot{E}_{u,t}^+\right]} \tag{2}$$

When any exergy term of the numerator exits the system without being used, the boundary of the system needs to be extended to the atmosphere and this exergy term becomes zero, but the corresponding exergy losses are still accounted for in $\dot{L}$ since the denominator has not changed. We can say that the related exergy loss is internalized and attributed to the system. For example, if the system is a combustion engine with a generator, its main service is to provide electricity, even though the cooling network has exergy that could potentially be used by others. However, if the exergy of the cooling network is not used but is destroyed in a cooling tower, it is automatically included in the exergy losses $\dot{L}$. Some authors [4, 5] subdivide $\dot{L}$ into:

$$\dot{L} = \dot{L}_D + \dot{L}_E$$

Where $\dot{L}_D$ includes the exergy destruction inside the strictly defined system and $\dot{L}_E$ includes the exergy destroyed between the system and the atmosphere. For example, if the designer knows that there is no use of the heat of an engine thermal cycle he should not only try to limit the exergy destruction inside the cycle itself but also design the cycle in such a way to try to minimize the heat exergy exiting the system.

Although different definitions of exergy efficiency can be found in the literature, a simple example will show the interest of having introduced the transformation exergy concept in its formulation. Take the heat exchanger of Figure 1 used to heat a substance (c) such as cold milk with another substance (h) e.g. hot water (two-stream heat transfer process with no phase-change).

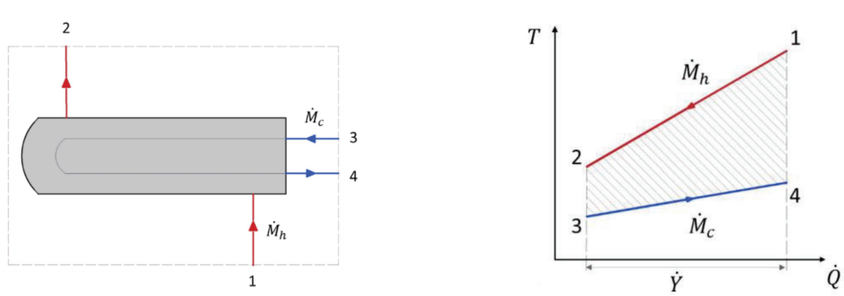

Fig. 1: Example of a heat transfer process through a heat exchanger
a) Schematic heat exchanger process, b) T-Q diagram of the hot and cold streams

https://doi.org/10.52202/069564-0012

The transformation (also called flow) energy ($\dot{Y}_h^+$) received by the system from the hot stream at a given temperature (T_h) is transferred through the heat exchanger to a cold stream ($\dot{Y}_c^-$) at a useful temperature (T_c). The above formulation of efficiency Eq. (2) reduces to:

$$\eta = \frac{\dot{E}_{yc}^-}{\dot{E}_{yh}^+} = \frac{\dot{M}_c\,(k_4 - k_3)}{\dot{M}_h\,(k_1 - k_2)} \leq 1 \tag{3}$$

Where $\dot{E}_{yh}^+$ is the exergy received from the hot stream (water), $\dot{E}_{yc}^-$ represents the one transferred to the cold stream (milk) through the heat exchanger and $k_j = h_j - T_a s_j$ represents the coenthalpy of the fluid in State j. That is coherent with the services provided and avoids discrepancies due to the fact that different thermodynamic references exist for the substances milk and water. Therefore an efficiency defined like:

$$\eta^* = \frac{\dot{M}_c\,k_4 + \dot{M}_h\,k_2}{\dot{M}_h\,k_1 + \dot{M}_c\,k_3} \tag{4}$$

would not be adequate to provide a coherent view of the exergy service to be provided. Furthermore, if two users refer to different thermodynamic databases they would get, for the same system, different values for the efficiency η^*. Such a confusion is avoided when using the definition η (Eq.3). It is interesting to note that the general exergy efficiency Definition (Eq.2) remains valid when a refrigeration service is demanded. In Figure 1 the milk could be cooled typically from the atmospheric temperature to a lower temperature by a water or brine achieving thus the required refrigeration service. Equation (3) remains unchanged since the cooled milk leaves with a higher exergy level while, on the other side the water loses some of its exergy from inlet to exhaust of the heat exchanger. This results from the fact that, as was shown in [1], the further the thermodynamic state of a substance, at a given pressure, is away from the dead state (atmosphere) the higher is its exergy level represented here by the coenthalpies.

It is not the same for the energy terms that do not change sign below and above the atmospheric temperature, with the exception of the heat losses to the atmosphere. The basic idea of the effectiveness is that all energy exchanges between a system and its atmospheric environment (taken from and/or released to the environment) are considered as losses. The general formulation of the ***effectiveness*** given in [1,2] (based on First Law only) in a similar manner to what has been done for the exergy efficiency would give:

$$\varepsilon = \frac{\dot{E}_w^- + \dot{Q}_i^- + Y_n^-}{\dot{E}_w^+ + \dot{Q}_i^+ + Y_n^+} \qquad T > T_a \tag{5}$$

This expression is indeed only valid for processes that take place above the atmospheric temperature and can take values between 0 and ∞. The effectiveness, as defined from the general Equation (Eq.2), is not an absolute quality indicator like the exergy efficiency is. It is still commonly used but mainly for relative comparison between system alternatives providing the same service. To illustrate this, let us consider the case of a simple heat pump for heating (a) and cooling (b) applications given in Figure 1.

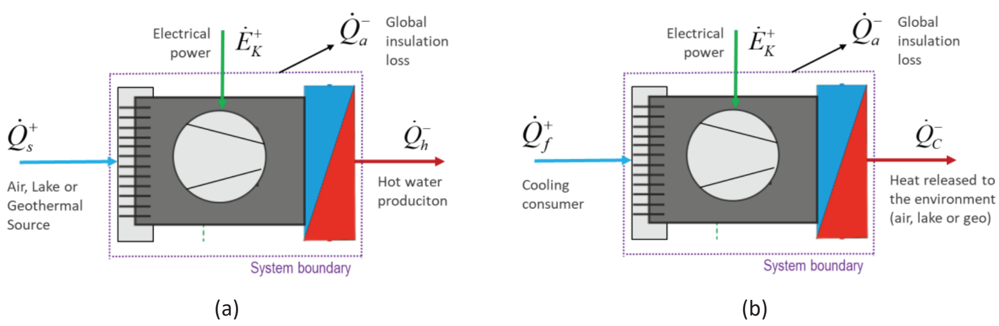

Fig. 2: Example of a vapor compression simple heat pump system: (a) for a heating, (b) for a cooling

For this case of a heat pump, the system boundary is defined by the closed cycle refrigerant (working fluid), so that the thermal energy exchanges with the exterior (with the user and/or the environment) are represented by thermal-heat only ($\dot{Q}$); there is no flow-energy ($\dot{Y}$) exchanged between the system and the exterior.

For the **heating** application, the system receives electricity ($\dot{E}_w^+ = \dot{E}_K^+$) and captures heat-energy ($\dot{Q}_s^+$) from the environment to deliver thermal heat ($\dot{Q}_h^-$) to a user for hot water production. Knowing that $\dot{Q}_s^+$ heat energy comes from the environment and is therefore "free" and thus not accounted for, the heating effectiveness $\boldsymbol{\varepsilon_h}$ obtained from Eq. (5) becomes:

$$\varepsilon_h = \frac{\dot{Q}_h^-}{\dot{E}_w^+} \geq 1 \tag{6}$$

Because the numerical value of this indicator is always higher than one and thus can no more semantically be called an "efficiency", this heating effectiveness is commonly called "coefficient of performance of heating (COP_h)".

The situation is different for the heat pump with **cooling** application of Figure 2b. The system receives electricity ($\dot{E}_w^+ = \dot{E}_K^+$) and provides cooling service to consumer by extracting energy ($\dot{Q}_f^+$) at low temperature and rejecting heat ($\dot{Q}_c^-$) to the environment at higher temperature. For this basic application, the cooling effectiveness $\boldsymbol{\varepsilon_f}$ is commonly defined in the literature by the following equation (Eq. 7) and is therefore not coherent for an effectiveness determined from the Eq. (5):

$$\varepsilon_f = \frac{\dot{Q}_f^+}{\dot{E}_w^+} \geq 1 \tag{7}$$

It can also take any numerical value between 0 and ∞, reason why it is commonly called *coefficient of performance of cooling* (COP_f) instead. For both applications of heating and cooling, the exergy efficiency based on Eq. (2) can be applied and the numerical value of these indicators are always lower than one, reason why we use the term of efficiency with the difference of the effectiveness:

$$\eta_h = \frac{\dot{E}_{qh}^-}{\dot{E}_w^+} < 1 \tag{8}$$

$$\eta_f = \frac{\dot{E}_{qf}^-}{\dot{E}_w^+} < 1 \tag{9}$$

While the exergy efficiency definition Eq. (2) is general and applies to all systems, it is not the case of the effectiveness (Eq. 5). The latter cannot be applied to all technologies and/or energy conversion systems. It is indeed only valid for processes that take place above the atmospheric temperature ($T \geq T_a$) and therefore must be adapted for refrigeration systems and also for any conversion systems providing simultaneously energy services for both heating and cooling.

3. New approach for a general expression of the effectiveness

3.1. A general expression of effectiveness applied to all technologies

To circumvent some of the difficulties to define a general expression for the effectiveness we propose the following expression in such a way to highlight $(\dot{U}_{0t})$ which represents the difference between the inlet services (work, gas, biomass…) expended for the system and the outlet services (work, heating and/or cooling) delivered by the system:

$$\varepsilon = \frac{\sum_t[\dot{U}_t^-]}{\sum_t[\dot{U}_t^+]} = 1 - \frac{\dot{U}_{0t}}{\sum_t[\dot{U}_t^+]} \tag{10}$$

Where $\sum_t[\dot{U}_t^-]$ is the total energy services provided or delivered by the system to users and $\sum_t[\dot{U}_t^+]$ represents the total energy services received by the system from utilities. Considering the different form of outlet energy services, the inlet/outlet ***energy services balance*** ($\dot{U}_{0t}$) can then clearly be formulated as follows:

$$\dot{U}_{0t} = \sum_t[\dot{U}_t^+] - \left(\sum_w[\dot{E}_w^-] + \sum_h[\dot{U}_h^-] + \sum_f[\dot{U}_f^+]\right) \tag{11}$$

Where each term (given in bracket) is a numerically positive value and the indices w, h and f refer to the types of energy services delivered or supplied by the system:

- $\sum_w[\dot{E}_w^-]$ mechanical or electrical energy services that can be **delivered** by the system
- $\sum_h[\dot{U}_h^-]$ hot services **delivered** by a system for a heating application
- $\sum_f[\dot{U}_f^+]$ cold services **supplied** by a system for a cooling application

This later term corresponds to a positive value because the main service in a cooling application is not to deliver heat but to capture heat at a temperature T_f lower than that of the ambient like in a fridge or any refrigeration system.

The inlet/outlet ***energy services balance*** ($\dot{U}_{0t}$) could also be obtained by using the First Law equation [1] and by separating the energy service terms from the total energy exchanged with the environment ($\dot{Q}_a^-$):

https://doi.org/10.52202/069564-0012

$$\sum_{k} \dot{E}^{+}_{w_k} + \sum_{i} \dot{Q}^{+}_{i} + \sum_{n} \dot{Y}^{+}_{n} = \dot{Q}^{-}_{a} \qquad (12)$$

$\dot{Q}^{-}_{a}$ may be exchanged in different forms such as heat transferred between the system and the environment, e.g. water, soil, air ($\dot{Q}^{-}_{0}$) and/or flow energy evacuated to the atmosphere, e.g. exhaust gases ($\dot{Y}^{-}_{0}$). The term $\dot{Q}^{-}_{0}$ may also include thermal losses due for example to the imperfection of the insulation.

A distinction is made because the terms $\dot{Q}^{-}_{0}$ *and* $\dot{Y}^{-}_{0}$ may include some valuable energies taking into account that the level of temperature of the fluid T_0 could be higher or lower than the ambient temperature T_a. This expression of the First Law Eq. (12) could also be reformulated by using the numerically positive values of energy services defined above and by highlighting the balance of energy exchanges with the environment:

$$\sum_{t}\left[\dot{U}^{+}_{t}\right] - \left(\sum_{w}\left[\dot{E}^{-}_{w}\right] + \sum_{h}\left[\dot{U}^{-}_{h}\right] + \sum_{f}\left[\dot{U}^{-}_{f}\right]\right) = \dot{Q}^{-}_{0} + \dot{Y}^{-}_{0} \qquad (13)$$

By subtracting the First law equation Eq. (13) from the inlet/outlet energy services equation Eq. (11), the inlet/outlet energy services balance ($\dot{U}_{0t}$) can lead to the following general equation, as function of the total energy exchanged between the system and its environment:

$$\dot{U}_{0t} = \dot{Q}^{-}_{0} + \dot{Y}^{-}_{0} - 2\sum_{f}\left[\dot{U}^{+}_{f}\right] \qquad (14)$$

This result shows clearly the difference between the heating and cooling application in term of the effectiveness. In a heating application, $\dot{U}_{0t}$ is equal to the total energy exchanged with the environment ($\dot{Q}^{-}_{a} = \dot{Q}^{-}_{0} + \dot{Y}^{-}_{0}$) but it is not the case for a cooling application where $\dot{U}^{+}_{f} \neq 0$. This result can be interpreted by the fact that the cold utility (or refrigerant) in the system is considered in the balance for both capturing heat from the user and providing cooling services to the same user.

Example of a simple heat pump system for heating application

Let us apply this definition to the previous examples, starting with the case of the heat pump for heating (Figure 2a). The energy service provided to customers corresponds to $\dot{Q}^{-}_{h}$ and the energy service received by the system is the electrical power ($\dot{E}^{+}_{k}$). There are no cooling services and no work or electric production for this application:

- $\sum_h\left[\dot{U}^{-}_{h}\right] = \dot{Q}^{-}_{h}$ Heating service
- $\sum_f\left[\dot{U}^{+}_{f}\right] = 0$ No cooling service for this application
- $\sum_w\left[\dot{E}^{-}_{w}\right] = 0$ No work or electrical production

There is no flow exchanged with environment ($\dot{Y}^{-}_{0} = 0$) and the heat exchanged with environment is represented by $\dot{Q}^{-}_{0} = \dot{Q}^{-}_{s}$. The energy balance based on the two equations of energy services Eq. (11) and energy exchange between the system and the environment Eq.(14) can be done as:

$$\dot{U}_{0t} = \dot{E}^{+}_{k} - \dot{Q}^{-}_{h} = \dot{Q}^{-}_{a} - \dot{Q}^{+}_{s} \qquad (16a)$$

Therefore, the effectiveness of the general Equation Eq. (10) corresponds to COP_h according to:

$$\varepsilon_h = 1 - \frac{\dot{E}^{+}_{k} - \dot{Q}^{-}_{h}}{\dot{E}^{+}_{K}} = \frac{\dot{Q}^{-}_{h}}{\dot{E}^{+}_{K}} = COP_h \qquad (16b)$$

$$\varepsilon_h = 1 + \frac{\dot{Q}^{+}_{s}}{\dot{E}^{+}_{K}} \qquad (16c)$$

A same result can be obtained by using Eq. 5 for this process of heating services which takes place above the atmospheric temperature.

Example a simple refrigeration system for cooling application:

In the case of the refrigeration unit of Figure 2b, the energy service provided to customers corresponds to $\dot{Q}^{+}_{f}$ and the energy service received by the system is the electrical power ($\dot{E}^{+}_{k}$). There are no heating services and no work or electric production for this application:

- $\sum_f\left[\dot{U}^{+}_{f}\right] = \dot{Q}^{+}_{f}$ Cooling service
- $\sum_h\left[\dot{U}^{-}_{h}\right] = 0$ No heating service for this application

- $\sum_w[\dot{E}_w^-] = 0$ No work or electrical production

There's no flow exchanged with environment ($\dot{Y}_0^- = 0$) and the heat exchanged with environment is represented by $\dot{Q}_0^- = \dot{Q}_c^-$. The energy balance based on the two equations of energy services Eq. (11) and energy exchange between the system and the environment Eq. (14) can be done as:

$$\dot{U}_{0t} = \dot{E}_k^+ - \dot{Q}_f^+ = \dot{Q}_c^- - 2\,\dot{Q}_f^+ \quad (17a)$$

Therefore, the effectiveness of the general Equation (Eq.10) corresponds to COP_f according to:

$$\varepsilon_f = 1 - \frac{\dot{E}_k^+ - \dot{Q}_f^+}{\dot{E}_K^+} = \frac{\dot{Q}_f^+}{\dot{E}_K^+} = COP_f \quad (17a)$$

$$\varepsilon_f = 1 - \frac{\dot{Q}_c^- - 2\,\dot{Q}_f^+}{\dot{E}_K^+} \quad (17a)$$

This result could not be obtained with Eq. (5) for this process of refrigeration that take place at a temperature level below the atmospheric temperature and also for any energy conversion processes providing simultaneously energy services for both heating and cooling (for example with heat pumps in cogeneration). For such cogeneration applications, the energy balance based on Eq. (11) and Eq. (14) can be done as:

$$\dot{U}_{0t} = \dot{E}_k^+ - \dot{Q}_h^- - \dot{Q}_f^+ = -2\,\dot{Q}_f^+ \quad (18a)$$

Therefore, the effectiveness of the general Equation Eq. (10) corresponds to $\varepsilon = \varepsilon_h + \varepsilon_f$ according to:

$$\varepsilon = 1 - \frac{\dot{E}_k^+ - \dot{Q}_h^- - \dot{Q}_f^+}{\dot{E}_K^+} = \frac{\dot{Q}_h^- + \dot{Q}_f^+}{\dot{E}_K^+} = \varepsilon_h + \varepsilon_f \quad (18b)$$

$$\varepsilon = 1 + \frac{2\,\dot{Q}_f^+}{\dot{E}_K^+} = 1 + 2\,\varepsilon_f \quad (18c)$$

Considering Eq. (18b) and Eq. (18c), we can deduce the well-known simple relation between the effectiveness for heating (ε_h) and cooling (ε_f):

$$\varepsilon_h = \varepsilon_f + 1 \quad (19)$$

3.2. A general relationship between energy and exergy indicators

From the equations of exergy efficiency (Eq.2) and effectiveness (Eq.10), a generic relationship can be deduced for the overall exergy losses and efficiencies of any energy conversion technologies as a function of the conventional performance indicators (engine efficiency, efficiency based on Lower Heating Value or on Higher Heating Value, heating coefficient of performance, cooling coefficient of performance) most commonly used by engineers to estimate various energy losses of a system:

$$\frac{\dot{L}}{\dot{U}_{0t}} = \frac{1-\eta}{1-\varepsilon}\,.\; R_{E/X} \quad (20)$$

Where $R_{E/X}$ represents the ratio of exergy services received by the system ($\sum_t[\dot{E}_{ut}^+]$) to the energy services expanded for the system ($\sum_t[\dot{U}_t^+]$). This ratio of exergy/energy is generally known for different sources and/or technologies. For electric vapor compression heat pumps, it is equal to the unity ($R_{E/X} = 1$); for thermal power cycle units, it corresponds to the Carnot factor related to the temperature level of the source ($R_{E/X} = 1 - T_a/\overline{T}_g$); For industrial fuels in combustion boilers, it can be determined by the coefficients given in [6, 7].

Such a general formulation of Eq. (20) can be applied to develop explicit relations between the global exergy losses ($\dot{L}$) and the external energy exchanges ($\dot{U}_{0t}$) with the environment (taken from and/or released to environment) and for any energy conversion technologies.

In fact, $\dot{U}_{0t}$ represents the difference between the inlet and the outlet energy services and can be determined or estimated in any case without any **knowledge of the entropy**. This can be very useful for practitioners to determine and know the exergy performance of any energy system based solely on energy balance terms and the effectiveness of the system.

https://doi.org/10.52202/069564-0012

4. Explicit relations of exergy losses for cogeneration technologies

4.1. Combined heat and power cogeneration systems

Any energy conversion system receiving heat-energy services ($\dot{Q}_g^+$) from a **hot** source at a given temperature level ($\bar{T}_g$) to both produce mechanical or electrical energy services ($\dot{E}_w^-$) and supply heat energy services ($\dot{Q}_h^-$) to a heating system at a lower temperature level ($\bar{T}_h$) is considered as a combined heat and power system.

There are many types of power-heat cogeneration plants based on steam turbine (ST) or organic fluid turbine (ORC), gas turbine (GT) and combined gas-steam cycle power plant technologies (CC), which can be found in particular in the chemical or food industry, in household waste incineration plants and of course in district heating thermal power stations. Figure 3 shows an example of a cogeneration plant with a power plant combining electricity and heat using the thermal energy of combustion gases in a boiler to generate steam which drives a power generation turbine. Steam extraction at an intermediate enthalpy level is used to provide energy services for the production of superheated water for a district heating network.

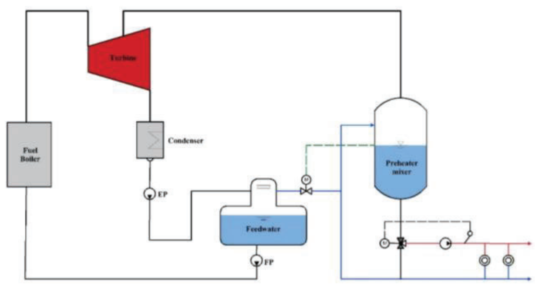

Figure 3: Schematic diagram of the example of steam turbine cycle cogeneration system

Other more decentralized cogeneration systems based on internal combustion engine or fuel cell technologies are used for small scale applications, e.g. in building.

For the example of the cogeneration system given in figure 3: the hot source is represented by the combustion gases ($\dot{Q}_g^+$) in the boiler, the output energy services are the net electricity production ($\dot{E}_w^- = \dot{E}_T^- - \dot{E}_P^+$, meaning the balance between the production in the turbine and the consumption in the various pumps) and the heat energy ($\dot{Q}_h^-$) obtained from the vapor extraction is to supply the heating circuit in the network.

Let us consider the general case of cogeneration where $\dot{Q}_a^-$ represents the heat losses to the atmosphere because of non-perfect insulated components, $\dot{Q}_0^-$ the quantity of energy released from the condenser and $\dot{Y}_0^-$ the exhaust energy out of the boiler. In this case, $\dot{U}_{0t}$ can be expressed by the following relations for steady state operation and based on Eq. (11) and Eq. (14):

$$\dot{U}_{0t} = \dot{Q}_g^+ - \left(\dot{E}_w^- + \dot{Q}_h^-\right) \qquad \dot{U}_{0t} = \dot{Q}_a^- + \dot{Q}_0^- + \dot{Y}_0^- \tag{21}$$

Thus, the effectiveness for cogeneration (ε) based on Eq. (10) can be given by:

$$\varepsilon = 1 - \frac{\dot{U}_{0t}}{\dot{Q}_g^+} = \frac{\dot{E}_w^- + \dot{Q}_h^-}{\dot{Q}_g^+} = \varepsilon_e + \varepsilon_h \tag{22}$$

Where $\varepsilon_e = \dot{E}_w^- / \dot{Q}_g^+$ represents the effectiveness for electricity production and $\varepsilon_h = \dot{Q}_h^- / \dot{Q}_g^+$ represents the effectiveness for heating. A same result can be obtained by using Eq. 5 for this process of cogeneration which takes place above the atmospheric temperature.

Same reasoning can be applied to determine the overall exergy efficiency (η) by the general equation (Eq. 2):

$$\eta = \frac{\dot{E}_w^- + \dot{E}_{qh}^-}{\dot{E}_{qg}^+} \tag{23}$$

Where $\dot{E}_{qg}^+$ represents the heat-exergy received by the system from the hot source and $\dot{E}_{qh}^-$ represents the heat-exergy provided by the system to the user for heating.

The exergy efficiency can then be determined by considering the Carnot factors related to the temperature levels of the hot source ($\bar{T}_g$) and of the heating system ($\bar{T}_h$):

$$\eta = \frac{\dot{E}_w^- + \left(1 - \frac{T_a}{\bar{T}_h}\right)\dot{Q}_h^-}{\left(1 - \frac{T_a}{\bar{T}_g}\right)\dot{Q}_g^+} = \frac{\varepsilon_e + \left(1 - \frac{T_a}{\bar{T}_h}\right)\varepsilon_h}{1 - \frac{T_a}{\bar{T}_g}} \tag{24}$$

In general, these temperature levels of the source and the heating system are initially known. Thus, improving the exergy performance of a cogeneration system shall involve also improving the effectiveness ($\varepsilon = \varepsilon_e + \varepsilon_h$) and therefore minimizing the total energy released to the environment Eq. (21). To illustrate this, we can apply the general expression for exergy losses Eq. (10) by replacing the exergy efficiency η by its value defined in Eq. (24). We can thus express the exergy losses in the most general form by the following relations:

$$\dot{L}_g = \left[1 + \frac{T_a}{1-\varepsilon}\left(\frac{\varepsilon_h}{\bar{T}_h} - \frac{1}{\bar{T}_g}\right)\right]\dot{U}_{0t} \tag{25}$$

Or on the basis of the system input energy:

$$\dot{L}_g = \left[1 - \varepsilon + T_a\left(\frac{\varepsilon_h}{\bar{T}_h} - \frac{1}{\bar{T}_g}\right)\right]\dot{Q}_g^+ \tag{26}$$

Two components of exergy losses can be distinguished: external exergy losses that are exactly equal to the energy releases to the environment (and therefore become zero for a system without energy exchange with the atmosphere) and internal exergy losses (exergy destruction) that are related to the temperature difference between source and user. The exergy losses by destruction in the system increase not only with the temperature difference but are lower the higher the source and user temperature levels are. The lower the temperature levels, the higher the losses for the same temperature difference.

Equation (24), Eq. (25) and Eq. (26) are general for any cogeneration system with or without combustion exhaust gases, using for example boiler, gas turbine or internal combustion engine technologies for which the overall effectiveness is less than unity (ε<1) taking into account the exhaust losses to the chimney ($\dot{Y}_0^-$) as well as the possible heat losses to the atmosphere ($\dot{Q}_a^-$) because of the high gas temperature. They can also be used for various particular or supposed cases for power generation only (powerplants), for small organic ranking cycles with cogeneration effectiveness close to unity (ε<1) and/or for simple heating applications with boilers or network substations with a perfectly insulated heat exchanger.

Case of **a powerplant:** Zero effectiveness for heating (ε_h=0) corresponds to a **power system** where the energy received from the hot source is only for producing mechanical or electrical energy and all energy that can be recovered from the cold source at lower temperature Th is destroyed to the atmosphere. This is the case for example for many **power-plants**. Thus, Eq. (24) and Eq. (26) become:

$$\eta = \frac{\varepsilon_e}{1 - T_a/\bar{T}_g} \qquad \dot{L}_g = \left[1 - \varepsilon_e - \frac{T_a}{\bar{T}_g}\right]\dot{Q}_g^+ \tag{27}$$

This is a well-known and particular relationship between the exergy efficiency of a power system, the effectiveness and the ideal effectiveness based on the Carnot factor.

Case of **a cogeneration system** based on **Organic Rankine Cycle:** Effectiveness of cogeneration equal to unity (ε=1) corresponds to a perfectly insulated–cogeneration system ($\dot{Q}_a^- = 0$) without any exhaust energy to the atmosphere ($\dot{Y}_0^- = 0$) and where the energy received from the hot source is totally used for producing mechanical or electrical energy and all residual and available energy of the cold source at lower temperature $\bar{T}_h$ is recovered for cogeneration. The external energy losses are zero ($\dot{U}_{0t} = 0$) and part of the exergy received is destroyed in the system. It is the case for example for cogeneration systems using **closed power-cycles e.g. with organic rankine cycles** which are supposed to be perfectly insulated. Equations (24) and Eq. (26) become:

$$\eta = \frac{1 - T_a/\bar{T}_h\,(1-\varepsilon_e)}{1 - T_a/\bar{T}_g} \qquad \dot{L}_g = \left[T_a\left(\frac{1-\varepsilon_e}{\bar{T}_h} - \frac{1}{\bar{T}_g}\right)\right]\dot{Q}_g^+ \tag{28}$$

Case of a **heating system with boiler or heat exchanger only:** Zero effectiveness for electricity (ε_e=0) corresponds to a **heating system** where the energy received from the hot source is only for supplying heat energy services ($\dot{Q}_h^-$) to a heating system at a lower temperature level ($\bar{T}_h$). Losses to the atmosphere ($\dot{U}_{0t}^-$) can take place in different forms, due for example to a defect in the insulation of system components ($\dot{Q}_a^- = 0$) or for exhaust gases ($\dot{Y}_0^- = 0$). This is the case for example for **heat transfer process with boiler or heat exchanger**. Thus, Eq. (26) and Eq. (28) become:

https://doi.org/10.52202/069564-0012

$$\eta = \frac{1 - T_a/\bar{T}_h}{1 - T_a/\bar{T}_g}\,\varepsilon_h \qquad \dot{L}_g = \left[1 - \varepsilon_h + T_a\left(\frac{\varepsilon_h}{\bar{T}_h} - \frac{1}{\bar{T}_g}\right)\right]\dot{Q}_g^+ \tag{29}$$

For all these above applications, one can show the important role of the temperature in the assessment of the overall exergy losses or efficiencies. To illustrate this, let us consider for example Eq. (31) by using the temperature difference ($\Delta T = \bar{T}_g - \bar{T}_h$) between the hot gases and the heating process. The following expressions are obtained:

$$\dot{L}_h = \left[(1 - \varepsilon_h)\left(1 - \frac{T_a}{\bar{T}_g}\right) + \varepsilon_h\,\frac{T_a}{\bar{T}_g}\,\frac{\Delta T}{\bar{T}_h}\right]\dot{Q}_g^+ \tag{30}$$

$$\eta_h = \varepsilon_h\left[1 - \frac{T_a}{\bar{T}_g - T_a}\,\frac{\Delta T}{\bar{T}_h}\right] \tag{31}$$

Two components of exergy losses can be distinguished: external heat transfer exergy losses that are rather related to the temperature level of the source (and therefore become zero for a system without energy exchange with the atmosphere) and internal heat losses (devaluation of energy) that are related to the temperature difference between source and user. The exergy losses by external transfer do increase with the temperature level of the source while the exergy losses by internal heat devaluation increase not only with the temperature difference but are lower the higher the source and user temperature levels are. The lower the temperature levels, the higher the exergy losses for the same temperature difference. The temperature therefore plays an important role in the assessment of the overall exergy losses and irreversibilities.

These expressions of exergy losses Eq. (30) and efficiency Eq. (31) for a heat transfer process not only classify but also define the qualities that any heat transfer system between a source and a user must have. Figures 4 show for a given effectiveness like in a heat exchanger ($\varepsilon_h = 1$) the influence of temperature difference between hot and cold streams on the specific exergy loss and efficiency in the heat transfer process, by considering respectively the parametric curves of the consumer's useful temperature ($\bar{T}_h$) and the temperature level of the hot source ($\bar{T}_g$). The specific exergy loss is given by the following $\alpha_L = \dot{L}_g/\dot{Q}_g^+$.

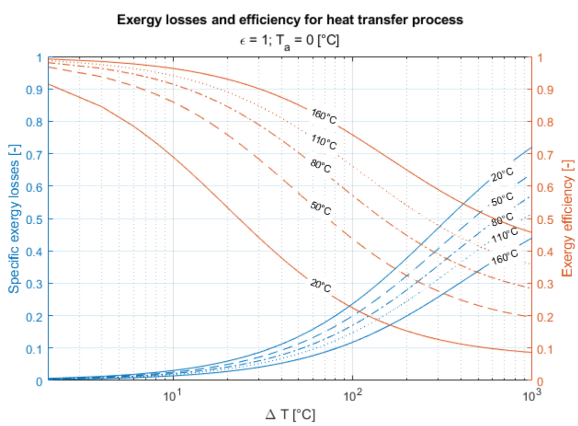

Fig. 5: The effect of the differential temperature on the exergy performance of a heat transfer process with parametric curves corresponding to the consumer's useful temperature ($\bar{T}_h$)

Zero heating effectiveness ($\varepsilon_h = 0$) corresponds to a heat transfer system where all energy received from the hot stream is destroyed to the atmosphere. It is the case for example for an **air-cooler**. Thus, Eq. (24) and Eq. (26) become:

$$\dot{L}_g = \left[\left(1 - \frac{T_a}{\bar{T}_g}\right)\right]\dot{Q}_g^+ \qquad \eta = 0 \tag{32}$$

In the case of a **perfectly insulated heat exchanger**, the effectiveness is assumed to be equal to unity ($\varepsilon_h = 1$). The external heat transfer losses are zero and part of the exergy received is destroyed by internal heat devaluation. Equation (32) and Eq. (33) become:

$$\dot{L}_g = \frac{T_a}{\bar{T}_g}\,\frac{\Delta T}{\bar{T}_g - \Delta T}\,\dot{Q}_g^+ \qquad \eta_t = 1 - \frac{T_a}{\bar{T}_g - T_a}\,\frac{\Delta T}{\bar{T}_g - \Delta T} \tag{33}$$

For the same temperature difference between the hot source (hot gases) and user (heating system), the heat exchange is better for a hot source at higher temperature. This is why, for the same temperature difference, the exergy losses in a heat exchanger for domestic hot water production or for LT heating system are greater than those in a substation of a HT district heating system.

This is also the case for example with heat exchangers for the production of domestic hot water, for heating in district heating substations between the superheated water and the water for heating or for heat production in water-cooled boilers.

4.2. Heating and cooling with heat pump cogeneration systems

Heating and/or cooling installations are often composed of many components that exchange energy with a thermodynamic circuit. These energy exchanges can take place in various forms, involving different processes, including vapor compression and absorption heat pump/refrigeration cycles most commonly investigated by engineers. Modern installations of heat pumps are used in cogeneration with combined heating and cooling. Figure 6 shows an example of such a system with a heat pump combining heating and cooling. It allows to capture low temperature energy from the environment $\dot{Q}_s^+$ (i.e. Lake or geothermal) at a certain temperature level ($\overline{T}_0$), to transfer heat to a fluid (principal circuit) via a heat exchanger and to use it as a cold source in a heat pump to provide a heating service ($\dot{Q}_h^-$) necessary for hot water production ($\overline{T}_h$). The cold fluid out of the heat pump is transported through a network of pipes to supply cooling energy to the consumer and returns back to the heat exchanger. The cooling service are represented with the flow or transformation energy service ($\dot{Y}_f^+$) at temperature level ($\overline{T}_f$).

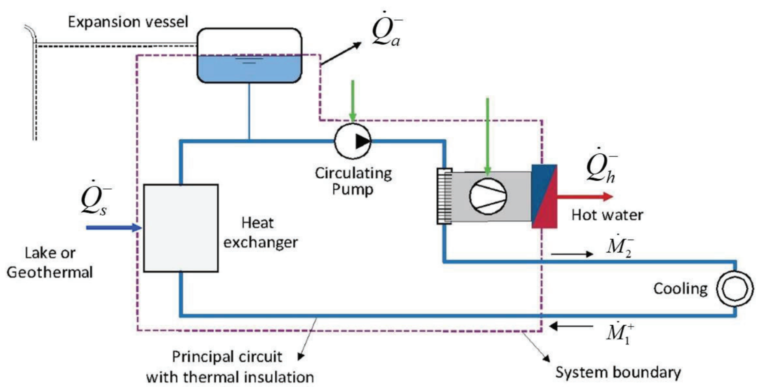

Fig. 6 *Example of a combined heating and cooling system*

The circulation of the heat transfer fluid is ensured by a circulating pump that overcomes the total pressure losses of the network. $\dot{E}_P^+$ represents the electrical power consumed by the circulating pump and $\dot{E}_K^+$ is the electrical power consumed by the heat pump. The input energy services are the net electricity consumption ($\dot{E}_w^+ = \dot{E}_P^+ + \dot{E}_K^+$). There is no work or electric production for this application:

- $\sum_f [\dot{U}_f^+] = \dot{Y}_f^+$ Cooling service, $\dot{Y}_f^+ = \dot{M}\left(h_{cz_1} - h_{cz_2}\right)$
- $\sum_h [\dot{U}_h^-] = \dot{Q}_h^-$ Heating service for this application
- $\sum_w [\dot{E}_w^-] = 0$ No work or electrical production

As the system is considered perfectly insulated (heat losses to the atmosphere are null) and there is no flow exchanged with environment ($\dot{Y}_0^- = 0$), the heat exchanged with environment is represented by $\dot{Q}_0^- = \dot{Q}_s^-$. The energy balance based on the two equations of energy services Eq. (11) and energy exchange between the system and the environment Eq. (14) can be done as:

$$\dot{U}_{0t} = \dot{E}_w^+ - \dot{Q}_h^- - \dot{Y}_f^+ \qquad \dot{U}_{0t} = -\left(\dot{Q}_s^+ + 2\,\dot{Y}_f^+\right) \tag{34}$$

Therefore, the effectiveness of the general Equation Eq. (10) corresponds to the total effectiveness according to:

$$\varepsilon = 1 - \frac{\dot{U}_{0t}}{\dot{E}_w^+} = \frac{\dot{Q}_h^- + \dot{M}\left(h_{cz_1} - h_{cz_2}\right)}{\dot{E}_w^+} = \varepsilon_h + \varepsilon_f \tag{35}$$

$$\varepsilon = 1 + \frac{\dot{Q}_s^+ + 2\,\dot{M}\left(h_{cz_1} - h_{cz_2}\right)}{\dot{E}_w^+} = \varepsilon_0 + 2\varepsilon_f \tag{36}$$

 https://doi.org/10.52202/069564-0012

Where $\varepsilon_h = \dot{Q}_h^- / \dot{E}_w^+$ and $\varepsilon_f = \dot{Y}_f^+ / \dot{E}_w^+$ respectively represent the effectiveness for heating and cooling and the term given by the following $\varepsilon_0 = 1 + \dot{Q}_s^+ / \dot{E}_w^+$ characterizes the amount of energy taken from the environment.

When considering equation Eq. 34, one also can demonstrate that ε_0 represents the difference between the effectiveness for heating and cooling ($\varepsilon_0 = \varepsilon_h - \varepsilon_f$). In the case of adiabatic components with no heat transfer losses and no energy withdrawal from the source ($\dot{Q}_s^+ = 0,\ \varepsilon_0 = 1$), we find the same relation Eq. (19) between the effectiveness of heating (ε_h) and cooling (ε_f) for a simple heat pump. This shows that the total effectiveness (ε) of such a combined heating/cooling cogeneration system also depends on this percentage of energy taken from the environment ($1 - \varepsilon_0$). These results based on Eq. (35) and Eq. (36) could not be obtained by using Eq. (5) for this process of cogeneration involving cooling services which take place at a temperature level below the atmospheric temperature.

Same reasoning can be applied to demonstrate that, the exergy performance of the system not only depends on the effectiveness of heating (ε_f) and cooling (ε_f) but also on the energy exchanged the environment ($1 - \varepsilon_0$). To illustrate that, let us first apply the general expressions of exergy efficiency Eq. (2) by considering the Carnot factors related to the temperature level of heating ($\bar{T}_h$) and cooling ($\bar{T}_f$). The overall exergy efficiency for heating/cooling cogeneration (η) is:

$$\eta = \frac{\dot{E}_{qh}^- + \dot{E}_{yf}^-}{\dot{E}_w^+} = \left(1 - \frac{T_a}{\bar{T}_h}\right) \varepsilon_h + \left(\frac{T_a}{\bar{T}_f} - 1\right) \varepsilon_f \tag{37}$$

Or then:

$$\eta = \varepsilon_0 - T_a \left(\frac{\varepsilon_h}{\bar{T}_h} - \frac{\varepsilon_f}{\bar{T}_f}\right) \tag{38}$$

Where $\dot{E}_w^+$ represents the work-exergy received by the system, $\dot{E}_{qh}^-$ represents the heat-exergy provided by the system to the user for heating and $\dot{E}_{yf}^-$ represents the flow-exergy provided by the system to the user for cooling.

By replacing the exergy efficiency η by its value defined in Eq. (20). We can thus express the exergy losses in the most general form by the following relations:

$$\dot{L}_g = \left[1 - \varepsilon_0 + T_a \left(\frac{\varepsilon_h}{\bar{T}_h} - \frac{\varepsilon_f}{\bar{T}_f}\right)\right] \dot{E}_w^+ \tag{39}$$

Two components of exergy losses can be distinguished: external exergy losses that are exactly equal to the energy taken from the environment and internal exergy losses (with irreversibilities) that are related to the temperature difference between heating and cooling consumers. The internal exergy losses in the system increase not only with the temperature difference but are lower the higher the temperature levels.

Equations (38) and (39) are general for any combined heating and cooling system using a heat pump technology with additional energy taken from the environment e.g. $\varepsilon_0 \neq 1$. For a particular case of a simple heat pump cogeneration application without any energy captured from the environment ($\dot{Q}_s^+ = 0, \varepsilon_0 = 1$), Eq. (38) and Eq. (39) become:

$$\eta = 1 - T_a \left(\frac{\varepsilon_h}{\bar{T}_h} - \frac{\varepsilon_f}{\bar{T}_f}\right) \qquad \dot{L}_g = \left[T_a \left(\frac{\varepsilon_h}{\bar{T}_h} - \frac{\varepsilon_f}{\bar{T}_f}\right)\right] \dot{E}_w^+ \tag{40}$$

Equations (38) and (39) can also be used for particular cases of a thermopump for heating only ($\varepsilon_0 = 1, \bar{T}_f = T_a$) or of a frigopump for cooling only ($\varepsilon_0 = 1, \bar{T}_h = T_a$). For example, by combining Eq. (20) and Eq. (38), we found the well-known and particular relationships between the exergy efficiency of a heat pump cycle, the coefficient of performance (effectiveness) and the ideal effectiveness based on the Carnot factor:

For a thermopump ($\bar{T}_f = T_a$):

$$\eta_h = \varepsilon_h \left(1 - \frac{T_a}{\bar{T}_h}\right) \qquad COP_h = \eta_h \cdot \frac{\bar{T}_h}{\bar{T}_h - T_a} \tag{41}$$

For a frigopump ($\bar{T}_h = T_a$):

$$\eta_f = \varepsilon_f \left(\frac{T_a}{\bar{T}_f} - 1\right) \qquad COP_f = \eta_f \cdot \frac{\bar{T}_f}{T_a - \bar{T}_f} \tag{42}$$

5. Conclusion

A general expression of the effectiveness applied to all technologies is proposed in this study. It is based on the First Law of thermodynamics by separating the energy service terms (inlet services in term of work, gas and/or biomass expended for a system and outlet services in term of work, heating and/or cooling delivered by the system) from the total energy exchanged with the environment. A clear difference between the heating and cooling processes is observed based on a general formula of energy services balance. In the case of a heating process, the difference between the inlet and outlet energy services is exactly equal to the total energy exchanged with the environment but it is not the case for a cooling application for which the energy exchanged with the refrigerant (cold utility) need properly to be considered in the balance by accounting for both the cooling services supplied by system to the user and the same quantity of energy received or captured by the system from the user.

Based on this new approach of effectiveness and the existing exergy efficiency formulation, a generic relationship between energy and exergy losses has been proposed. It allows to provide simple and generic expressions of exergy losses and efficiencies of any energy systems in relation to the conventional performance indicators (thermal efficiency, heating coefficient of performance, cooling coefficient of performance) most commonly used by engineers to estimate the various losses of a system. Such a method has been applied for complex cogeneration systems (combining power, heating and/or cooling) but also for single energy components, e.g. boilers, heat exchangers and air-coolers to show the benefit. The overall exergy efficiency and losses of such systems explicitly are given in function of the effectiveness.

Main results and advantages of such an approach are: Having a unique formula of effectiveness for both heating and cooling applications and creating the link between the known performance indicators; Providing a general relationship between exergy efficiency and effectiveness of any system; Performing the exergy analysis of common installations without any knowledge of entropy; Providing simple and generic relations of exergy efficiencies and losses and/or estimating the detailed distribution of the exergy losses of a system according to the different forms of energy losses (heat losses, exhaust gases, heat energies taken from or released to the environment, e.g. water, soil and air).

These can be very useful for practitioners to determine and know the exergy performance of any energy system based solely on energy balance terms and thus the effectiveness of the system.

Nomenclature

Roman symbols

- E work energy, exergy, J
- k co-enthalpy, J
- L global exergy losses, J
- $\dot{M}$ mass flow rate, kg/s
- Q heat energy, J
- Y flow or transformation energy, J
- T temperature, K
- U utility, energy service, J

Greek symbols

- η exergy efficiency
- ε effectiveness

Subscripts and superscripts

- a ambient, atmosphere
- f cooling service
- h heating service
- K compressor

References

[1] Favrat D., Kane M., Clear paths to teach exergy. Proc. of ECOS2023

[2] Borel, L., Favrat, D., (2010). Thermodynamics and energy systems analysis. EPFL Press,

[3] Adihou Y. (2022). Modélisation et optimisation exergétique des réseaux thermiques basse température. Thèse Doctorat USMB

[4] Bejan, A., Tsatsaronis, G., Moran, M.J., (1995). Thermal Design and Optimization. John Wiley & Sons

[5] Moran M.J., Shapiro H.N., Boettner D.D, Bailey M.B. Fundamentals of engineering thermodynamics. Wiley 9th edition 2018

[6] Szargut J., Moris D.R., Steward F.R, (1979). Exergy analysis of thermal, chemical, and metallurgical processes. Hemisphere &Springer-Verlag, 397 pages, ISBN 0-89116048-5

[7] Kotas, T.J., (1985). The Exergy Method of Thermal Plant Analysis. Elsevier ,

https://doi.org/10.52202/069564-0012